The field equations for gravitation and electromagnetism

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***Abstract* — Aim: The development of a unified field theory for electromagnetism and gravitation while no material source terms are taken into account in the field equations of unified field theory is still an issue to be solved.**
Methods: The usual tensor calculus rules were used.
Results: The gravitational and the electromagnetic field are successfully joined into one single hyper-field whose equations are determined only by the metric field $g_{\mu\nu}$ itself.
Conclusion: The theory of gravitation and electromagnetism of Einstein and Maxwell are unified and described exclusively in terms of the metric tensor $g_{\mu\nu}$.

Keywords — ***Gravitation, Electromagnetism, Einstein's field equations Unified field theory, Causality, Cause, Effect, Necessary condition.***

I. Introduction

The many short-lived attempts to unify the electromagnetic and gravitational fields together with other fundamental interactions within one conceptual and formal framework has been the focus of much present research but has not yet met with ultimate success. In point of fact, the unity of nature as the foundation for the unity of science allow us to hope to find the trail of the hidden in order to unify all the fundamental interactions, including gravitation[36] and electromagnetism, in a single theoretical framework – a unified field theory. However, so far all attempts to tie the known fundamental forces of nature and to unify[35], [37] the general[16], [19]–[21] theory of relativity[15], which describes the gravitational[2] field, with quantum mechanics made their incompatibility more or less more apparent. Even Einstein's efforts devoted to the search for a '**unified field theory**'[17], "**a generalization of the theory of the gravitational field**"[25], were in vain[27]. In order to reach new horizons on this issue in question, a lot can been learned even from failed attempts at unification and much more might be learned in the future. In brief, past experience has shown that due to the weakness of the gravitational interaction to encounter primarily an empirical-inductive method at unification with the goal to join the gravitational and electromagnetic field into a new field appears not impossible but hardly implausible. Therefore, in this attempt at unification prefers the deductive-hypothetical method. And it's with this goal in mind that greater attention will be directed in particular to the development of a new and systematic approach to the problem of unification. Fully in line with this is the fact that all tensors of the general theory of relativity, including the Ricci tensor[38] $R_{\mu\nu}$ can meanwhile be presented in terms of the metric tensor[9] $g_{\mu\nu}$ directly. However, to sum[5] up, what do we do as a consequence of it? In the following, it is necessary to put some focus on the development and building of a unified field theory on the basis of a metric tensor $g_{\mu\nu}$.

II. Material and Methods

2.1 Definitions

Evidently there several features of general relativity (particles as singularities of a field) including experimental[41] confirmation which have made the majority of physicists to believe that general relativity theory is correct. However, physicists have not yet figured out how to unify general relativity with all other interactions and the search for a quantum theory of gravity is still without a breakthrough in this field.

Definition 2.1 (**Anti tensor**). *Let $a_{\mu\nu}$ denote a co-variant (lower index) second-rank tensor. Let $b_{\mu\nu}$ denote another co-variant second-rank et cetera. Let $E_{\mu\nu}$ denote the sum of these co-variant second-rank tensors. Let the relationship $a_{\mu\nu} + b_{\mu\nu} + ... \equiv E_{\mu\nu}$ be given. A co-variant second-rank anti tensor [9] of a tensor $a_{\mu\nu}$ denoted in general as $\underline{a}_{\mu\nu}$ is defined*

$$\begin{aligned} \underline{a}_{\mu\nu} &\equiv E_{\mu\nu} - a_{\mu\nu} \\ &\equiv b_{\mu\nu} + ... \end{aligned} \tag{1}$$

Let $a^{\mu\nu}$ denote a contra-variant (upper index) second-rank tensor. Let $b^{\mu\nu}$ denote another contra-variant (upper index) second-rank et cetera. Let $E^{\mu\nu}$ denote the sum of these contra-variant (upper index) second-rank tensors. Let the relationship $a^{\mu\nu} + b^{\mu\nu} + ... \equiv E^{\mu\nu}$ be given. A co-variant second-rank anti tensor of a tensor $a^{\mu\nu}$ denoted in general as $\underline{a}^{\mu\nu}$ is defined

$$\begin{aligned} \underline{a}^{\mu\nu} &\equiv E^{\mu\nu} - a^{\mu\nu} \\ &\equiv b^{\mu\nu} + ... \end{aligned} \tag{2}$$

Let ${a_{\mu}}^{\nu}$ denote a mixed second-rank tensor. Let ${b_{\mu}}^{\nu}$ denote another mixed second-rank et cetera. Let ${E_{\mu}}^{\nu}$ denote the sum of these mixed second-rank tensors. Let the relationship ${a_{\mu}}^{\nu} + {b_{\mu}}^{\nu} + ... \equiv {E_{\mu}}^{\nu}$ be given. A mixed second-rank anti tensor of a tensor ${a_{\mu}}^{\nu}$ denoted in general as ${\underline{a}_{\mu}}^{\nu}$ is defined

$$\begin{aligned} {\underline{a}_{\mu}}^{\nu} &\equiv {E_{\mu}}^{\nu} - {a_{\mu}}^{\nu} \\ &\equiv {b_{\mu}}^{\nu} + ... \end{aligned} \tag{3}$$

Symmetric tensors of rank 2 may represent many physical properties objective reality. A co-variant second-rank tensor $a_{\mu\nu}$ is symmetric if

$$a_{\mu\nu} \equiv a_{\nu\mu} \tag{4}$$

However, there are circumstances, where a tensor is anti-symmetric. A co-variant second-rank tensor $a_{\mu\nu}$ is anti-symmetric if

$$a_{\mu\nu} \equiv -a_{\nu\mu} \tag{5}$$

Thus far, there are circumstances were an anti-tensor is identical with an anti-symmetrical tensor.

$$a_{\mu\nu} \equiv E_{\mu\nu} - b_{\mu\nu} + ... \equiv E_{\mu\nu} - \underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{6}$$

Under conditions where $E_{\mu\nu}$ = 0, an anti-tensor is identical with an anti-symmetrical tensor or it is

$$-\underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{7}$$

However, an anti-tensor is not identical with an anti-symmetrical tensor as such.

Definition 2.2 (**The Ricci tensor $R_{\mu\nu}$**). *Let $R_{\mu\nu}$ denote the Ricci tensor[38] of 'Einstein's general theory of relativity'[16], a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field.*

$$\begin{aligned} R_{\mu\nu} &\equiv \underbrace{\left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right)}_{a_{\mu\nu}+b_{\mu\nu}} + \underbrace{\left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})\right)}_{c_{\mu\nu}+d_{\mu\nu}} \\ &\equiv (a_{\mu\nu} + b_{\mu\nu}) + (c_{\mu\nu} + d_{\mu\nu}) \\ &\equiv (a_{\mu\nu} + c_{\mu\nu}) + (b_{\mu\nu} + d_{\mu\nu}) \\ &\equiv (a_{\mu\nu}) + (+b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}) \\ &\equiv (b_{\mu\nu}) + (+a_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}) \\ &\equiv (c_{\mu\nu}) + (+a_{\mu\nu} + b_{\mu\nu} + d_{\mu\nu}) \\ &\equiv (d_{\mu\nu}) + (+a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu}) \\ &\equiv a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} \end{aligned} \tag{8}$$

Definition 2.3 (**The Ricci scalar R**). *Under conditions of Einstein's general[16], [18]–[21] theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric is determined at each point in space-time by lamda Λ and anti-lamda[4] $\underline{\Lambda}$ as*

$$R \equiv g^{\mu\nu} \times R_{\mu\nu} \equiv (\Lambda) + (\underline{\Lambda}) \tag{9}$$

A Ricci scalar curvature R which is positive at a certain point indicates that the volume of a small ball about the point has smaller volume than a ball of the same radius in Euclidean space. In contrast to this, a Ricci scalar curvature R which is negative at a certain point indicates that the volume of a small ball is larger than it would be in Euclidean space. In general it is

$$R \times g_{\mu\nu} \equiv (\Lambda \times g_{\mu\nu}) + (\underline{\Lambda} \times g_{\mu\nu}) \tag{10}$$

The cosmological constant can also be written algebraically as part of the stress–energy tensor, a second order tensor as the source of gravity (energy density).

Definition 2.4 (**The stress-energy tensor of matter / energy E$_{\mu\nu}$**). *The stress-energy tensor of matter / energy $E_{\mu\nu}$ is determined in detail as follows.*

$$\begin{aligned}
E_{\mu\nu} &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\
&\equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \\
&\equiv \underbrace{\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)}_{E} \times g_{\mu\nu} \\
&\equiv E \times g_{\mu\nu} \\
&\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \\
&\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \\
&\equiv R_{\mu\nu} - \underline{E}_{\mu\nu} \\
&\equiv a_{\mu\nu} + b_{\mu\nu}
\end{aligned} \tag{11}$$

In our understanding, the stress-energy tensor of the electromagnetic field (b$_{\mu\nu}$) is equivalent to the portion of the stress-energy tensor of matter / energy (E$_{\mu\nu}$)due to the electromagnetic field where where T$_{\mu\nu}$ "denotes the co-variant energy tensor of matter"[24, p. 88]. In other words, there is no third tensor between the stress-energy tensor of the electromagnetic field (b$_{\mu\nu}$) and the tensor of ordinary matter or matter in the narrower sense (a$_{\mu\nu}$), **a third tensor is not given, tertium non datur**! "*Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense.* "[24, p. 93]

Electromagnetic field b$_{\mu\nu}$

Ordinary matter a$_{\mu\nu}$

Fig. 1. Ordinary matter and electromagnetic field.

Vranceanu[43] is elaborating on this issue too. In other words, the energy tensor T$_{kl}$ is treated by Vranceanu as the sum of two tensors one of which is due to the electromagnetic field (b$_{\mu\nu}$).

"On peut aussi supposer que le tenseur d'énergie T$_{kl}$ soit la somme de deux tenseurs dont un dû au champ électromagnétique . . . "[43]

Translated into English: '*One can also assume that the energy tensor T$_{kl}$ be the sum of two tensors one of which is due to the electromagnetic field.*'In this context, it is necessary to make a distinction between the relationship between ordinary matter and electromagnetic field and matter and gravitational field. Our understanding of the relationship between matter and gravitational field is impacted by Einstein's train of thoughts on this relationship. Einstein wrote:

> "Wir unterscheiden im folgenden zwischen 'Gravitationsfeld 'und 'Materie 'in dem Sinne, daß alles außer dem Gravitationsfeld als 'Materie 'bezeichnet wird, also nicht nur die 'Materie 'im üblichen Sinne, sondern auch das elektromagnetische Feld."[see 16, p. 802/803]

Einstein's position translated into English:

We make a distinction hereafter between 'gravitational field 'and 'matter'in this way that everything but the gravitational field is termed as 'matter'that is to say not only 'matter'in the ordinary sense, but the electromagnetic field as well.

Following Einstein, matter and gravitational field are complementary to each other. In other words, **all but gravitational field is matter** or there is no third between matter and gravitational field, a third is not given. Matter is the complementary of gravitational field and vice versa. The gravitational field is the complementary of matter. Again, each of both is equally the complementary of its own other. The following figure may illustrate this relationship in more detail.

Gravitational field

Matter

Fig. 2. Matter and gravitational field.

Definition 2.5 (**Laue's scalar T**). *Max von Laue (1879-1960) proposed the meanwhile so called Laue scalar[32] (criticised [26] by Einstein) as the contraction of the the stress–energy momentum tensor $T_{\mu\nu}$ denoted as T and written without subscripts or arguments. Under conditions of Einstein's general[16], [18]–[21] theory of relativity, it is*

$$T \equiv g^{\mu\nu} \times T_{\mu\nu} \tag{12}$$

where $T_{\mu\nu}$ "denotes the co-variant energy tensor of matter"[24, p. 88]. In other words, "Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense."[24, p. 93]

Definition 2.6 (**Einstein's field equations**). *Let $R_{\mu\nu}$ denote the Ricci tensor[38] of 'Einstein's general theory of relativity'[16], a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. Let R denote the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold. Ricci scalar curvature is the contraction of the Ricci tensor and is written as R without subscripts or arguments. Let Λ denote the Einstein's cosmological constant. Let $\underline{\Lambda}$ denote the "**anti cosmological constant**"[4]. Let $g_{\mu\nu}$ metric tensor of Einstein's general theory of relativity. Let $G_{\mu\nu}$ denote Einstein's curvature tensor. Let $\underline{G}_{\mu\nu}$ denote the "**anti tensor**"[6] of Einstein's curvature tensor. Let $E_{\mu\nu}$ denote stress-energy tensor of energy. Let $\underline{E}_{\mu\nu}$ denote tensor of non-energy, the anti-tensor of the stress-energy tensor of energy. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field. Let c denote the speed of the light in vacuum, let γ denote Newton's gravitational "**constant**"[3], [4], [6], [7]. Let π denote the number pi. Einstein's field equation, published by Albert Einstein[20] for the first time in 1915, and finally 1916[16] but later with the "**cosmological constant**"[18], [19], [21] term are determined as*

$$R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \equiv E_{\mu\nu} \tag{13}$$

Definition 2.7 (The tensor of non-energy). *Under conditions of Einstein's general[16], [18]–[21] theory of relativity, the tensor of non-energy or the anti tensor of the stress energy tensor is defined/derived/determined as follows:*

$$\begin{aligned} \underline{E}_{\mu\nu} &\equiv R_{\mu\nu} - \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \\ &\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}\right) \\ &\equiv c_{\mu\nu} + d_{\mu\nu} \end{aligned} \tag{14}$$

Definition 2.8 (Einstein's tensor G$_{\mu\nu}$). *Under conditions of Einstein's general[16], [18]–[21] theory of relativity, Einstein's tensor $G_{\mu\nu}$ is defined/derived/determined as follows:*

$$\begin{aligned} G_{\mu\nu} &\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \\ &\equiv \left(S - \frac{R}{2}\right) \times g_{\mu\nu} \\ &\equiv a_{\mu\nu} + c_{\mu\nu} \end{aligned} \tag{15}$$

Definition 2.9 (The anti Einstein's curvature tensor or the tensor or non-curvature). *Under conditions of Einstein's general[16], [18]–[21] theory of relativity, the tensor of non-curvature is defined/derived/determined as follows:*

$$\begin{aligned} \underline{G}_{\mu\nu} &\equiv R_{\mu\nu} - G_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right)\right) \\ &\equiv \left(\frac{R}{2}\right) \times g_{\mu\nu} \\ &\equiv b_{\mu\nu} + d_{\mu\nu} \end{aligned} \tag{16}$$

Definition 2.10 (The stress-energy tensor of ordinary matter a$_{\mu\nu}$). *Under conditions of Einstein's general[16], [18]–[21] theory of relativity, the stress-energy tensor of ordinary matter $a_{\mu\nu}$ is defined/derived/determined as follows:*

$$\begin{aligned} a_{\mu\nu} &\equiv a_{\mu\nu} + b_{\mu\nu} - b_{\mu\nu} \\ &\equiv a_{\mu\nu} + c_{\mu\nu} - c_{\mu\nu} \\ &\equiv a_{\mu\nu} - d_{\mu\nu} + d_{\mu\nu} \\ &\equiv R_{\mu\nu} - b_{\mu\nu} - c_{\mu\nu} - d_{\mu\nu} \end{aligned} \tag{17}$$

or equally as

$$\begin{aligned} a_{\mu\nu} &\equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) - b_{\mu\nu} \\ &\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) - b_{\mu\nu} \\ &\equiv R_{\mu\nu} - (R \times g_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) + d_{\mu\nu} \end{aligned} \tag{18}$$

or as

$$\begin{aligned} a_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \\ - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \end{aligned} \tag{19}$$

Definition 2.11 (**The tensor $b_{\mu\nu}$**). *The co-variant stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, is of order two and its components can be displayed by a 4 × 4 matrix too. Under conditions of Einstein's general[16], [18]–[21] theory of relativity, the tensor $b_{\mu\nu}$ denotes the stress-energy tensor of the electromagnetic field[29, p. 38] expressed more compactly and in a coordinate-independent form as*

$$\begin{aligned} b_{\mu\nu} &\equiv \left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu\,c}\times F_{\nu\,d}\times g^{cd}\right)-\left(\frac{1}{4}\times g_{\mu\nu}\times F_{de}\times F^{de}\right)\right)\right)\\ &\equiv \frac{1}{4\times 4\times\pi}\times\left(\left(F_{\mu\,c}\times F^{\mu\,c}\right)-\left(F_{de}\times F^{de}\right)\right)\times g_{\mu\nu}\\ &\equiv\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-d_{\mu\nu}\end{aligned} \tag{20}$$

where F_{de} is called the (traceless) Faraday/electromagnetic/field strength tensor.

Definition 2.12 (**The tensor $c_{\mu\nu}$**). *Under conditions of Einstein's general[16], [18]–[21] theory of relativity, the tensor of non-momentum and curvature is defined / derived / determined as follows:*

$$c_{\mu\nu}\equiv b_{\mu\nu}-\left(\Lambda\times g_{\mu\nu}\right) \tag{21}$$

Definition 2.13 (**The tensor $d_{\mu\nu}$ (neither curvature nor momentum)**). *Under conditions of Einstein's general[16], [18]–[21] theory of relativity, the tensor of neither curvature nor momentum is defined/derived/determined as follows:*

$$\begin{aligned} d_{\mu\nu} &\equiv\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-b_{\mu\nu}\\ &\equiv\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-\left(\Lambda\times g_{\mu\nu}\right)-c_{\mu\nu}\end{aligned} \tag{22}$$

There may exist circumstances where this tensor indicates pure vacuum, the space devoid of any matter.

Table 1 provides an overview of the definitions of the four basic[5], [6] fields of nature.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Tabelle 1: Einstein field equations and the four basic fields of nature

Definition 2.14 (**The inverse metric tensor $g^{\mu\nu}$ and the metric tensor $g_{\mu\nu}$**). *Under conditions of Einstein's general[16], [18]–[21] theory of relativity, it is [16, p. 796]:*

$$g_{\mu\nu}\times g^{\mu\nu}\equiv +4 \tag{23}$$

or

$$\frac{1}{g_{\mu\nu}\times g^{\mu\nu}}\equiv\frac{1}{4} \tag{24}$$

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other.

Einstein's point of view is that "... in the general theory of relativity ... must be ... the tensor $g_{\mu\nu}$ of the gravitational potential"[24, p. 88]

Definition 2.15 (**Index raising**). *For an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices[31] raises each index. In simple words, it is*

$$F^{\left(\begin{smallmatrix}1&3\\ \mu&c\end{smallmatrix}\right)}\equiv g^{\left(\begin{smallmatrix}1&2\\ \mu&\nu\end{smallmatrix}\right)}\times g^{\left(\begin{smallmatrix}3&4\\ c&d\end{smallmatrix}\right)}\times F_{\left(\begin{smallmatrix}\nu&d\\ 2&4\end{smallmatrix}\right)} \tag{25}$$

or more professionally

$$F^{\mu\, c} \equiv g^{\mu\nu} \times g^{cd} \times F_{\nu\, d} \tag{26}$$

2.2 Axioms

2.3 Axioms in general

Axioms [28] and rules which are chosen carefully can be of use to avoid logical inconsistency and equally preventing science from supporting particular ideologies. Rightly or wrongly, long lasting advances in our knowledge of nature are enabled by suitable axioms [12] too. Einstein himself brings it again to the point. [see 23, p. 17]

> “**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**.”[see 23, p. 17]

Einstein’s previous position now been translated into English: *The truly great advances in our understanding of nature originated in a manner almost diametrically opposed to induction.* It is worth mentioning in this matter, Einstein himself advocated especially basic laws (axioms) and conclusions derived from the same as a main logical foundation of any ‘theory’.

> “**Grundgesetz (Axiome)**
> und
> **Folgerungen**
> zusammen bilden das was man
> **eine ‘Theorie’**
> nennt. ”[see 23, p. 17]

Albert Einstein’s (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a ‘theory’* has still to get round. However, it is currently difficult to ignore completely these historical and far reaching words of wisdom. The same taken more seriously and put into practice, will yield an approach to fundamental scientific problems which is more creative and sustainably logically consistent. Historically, Aristotle himself already cited **the law of excluded middle** and **the law of contradiction** as examples of axioms. However, **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally as the most simple axiom of science. Something which is really just itself is equally different from everything else. In point of fact, is such an equivalence which everything has to itself inherent or must the same be constructed by human mind and consciousness. Following Gottfried Wilhelm Leibniz (1646-1716):

> “**Chaque chose est ce qu’elle est.**
> **Et dans autant d’exemples qu’on voudra**
> **A est A, B est B**. ”[see 33, p. 327]

or **A = A**, **B = B** or **+1 = +1**. In this context, **lex contradictionis**, the negative of lex identitatis, or **+0 = +1** is of no minor importance too.

2.3.1 Axiom I. Lex identitatis

To say that +1 is identical to +1 is to say that both are the same.
AXIOM 1. LEX IDENTITATIS.

$$(g_{\mu\nu} \times g^{\mu\nu}) - 3 \equiv +1 \tag{27}$$

or

$$+1 \equiv +1 \tag{28}$$

However, even such a numerical identity which seems in itself wholly unproblematic, for it indicates just to a relation which something has to itself and nothing else, is still subject to controversy. Another increasingly popular view is that the same numerical identity implies the controversial view that we are talking about two different numbers +1. The one +1 is on the left side on the equation, the other +1 is on the right side of an equation. The basicness of the relation of identity implies the contradiction too while circularity is avoided. In other words, how can the same +1 be identical with itself and be equally different from itself? We may usefully state that identity is an utterly problematic notion and might be the most troublesome of all.

2.3.2 Axiom II. Lex contradictionis

Axiom 2. Lex contradictionis.

$$(g_{\mu\nu} \times g^{\mu\nu} - 3) - (g_{\mu\nu} \times g^{\mu\nu} - 3) \equiv +1 \tag{29}$$

or

$$+0 \equiv +1 \tag{30}$$

A considerable obstacle to understanding contemporary usage of the term contradiction, however, is that contradiction does not seem to be a unitary one. How can something be both, itself (**a path is a straight line** from the standpoint of a co-moving observer at a certain point in space-time) **and** the other of itself, its own opposition (**the same path is not a straight line**, the same path is curved, from the standpoint of a stationary observer **at a certain point in space-time**) [8]. We may simply deny the existence of objective or of any other contradictions. Furthermore, even if it remains especially according to Einstein's special theory of relativity that it is not guaranteed that the notion of an absolute contradiction is justified, Einstein's special theory of relativity insist that contradictions are objective and real. That this is so highlights the fact that from the standpoint of a co-moving observer, under certain circumstances, **a path is a straight line** and nothing else. However, under the same circumstances of special theory of relativity where the relative velocity v > 0, from the standpoint of a stationary observer **the same path is a not a straight line, the path is curved**. The justified question is, why should and how can an identical be a contradictory too?

2.3.3 Axiom III. Lex negationis

Axiom 3. Lex negationis.

$$\neg(+0) \times 0 = (+1) \tag{31}$$

where $\neg$ denotes the (natural "**determinatio negatio est**"[see 40, p. 634] / logical[11]) process of negation.

III. Results

Theorem 3.1 (Einstein's tensor $G_{\mu\nu}$ is determined by the tensor of ordinary matter $a_{\mu\nu}$)**.** *Depending upon circumstances, Einstein's tensor $G_{\mu\nu}$ can be decomposed in many different ways. However, the tensor of ordinary matter $a_{\mu\nu}$ is one determining part of Einstein's tensor.*
Claim.

***If** the premise*

$$\underbrace{+1 = +1}_{(Premise)} \tag{32}$$

*is true, **then** the conclusion*

$$a_{\mu\nu} + \underline{a}_{\mu\nu} \equiv G_{\mu\nu} \tag{33}$$

is also true, the absence of any technical errors presupposed.

Proof. The premise

$$(+1) = (+1) \tag{34}$$

of this theorem is true. Multiplying this premise by $\left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right)$, the stress-energy tensor of matter, it is

$$1 \times \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \equiv 1 \times \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \tag{35}$$

or

$$\left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \tag{36}$$

Equation 36 changes according to equation 11 (defintion 2.4)

$$(G_{\mu\nu} + (\Lambda \times g_{\mu\nu})) \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \tag{37}$$

Equation 11 (definition 2.4) is $+a_{\mu\nu} + b_{\mu\nu} \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right)$. Therefore, equation 37 can be rearranged as

$$G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \equiv +a_{\mu\nu} + b_{\mu\nu} \tag{38}$$

Einstein's tensor $G_{\mu\nu}$ follows as

$$G_{\mu\nu} \equiv +a_{\mu\nu} + b_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{39}$$

We define an anti tensor of the tensor $a_{\mu\nu}$, denoted by $\underline{a}_{\mu\nu}$, according to definition 2.1 as $\underline{a}_{\mu\nu} \equiv +b_{\mu\nu} - (\Lambda \times g_{\mu\nu})$. Equation 39 simplifies further as

$$G_{\mu\nu} \equiv \mathbf{+a}_{\mu\nu} + \underline{a}_{\mu\nu} \tag{40}$$

In other words, the stress-energy tensor of ordinary matter $+a_{\mu\nu}$ is a determining part of Einstein's curvature tensor $G_{\mu\nu}$. □

Remark 3.1. *Theorem 3.1 has been able to provide evidence that Einstein's tensor $G_{\mu\nu}$ is determined by the tensor of ordinary matter $a_{\mu\nu}$ as $G_{\mu\nu} \equiv \boldsymbol{+a}_{\mu\nu} + \underline{a}_{\mu\nu}$. Equation 40 (theorem 3.1) is demanding equally a concrete structure of an anti-tensor of ordinary matter, denoted as $\underline{a}_{\mu\nu}$, in the following form: $\underline{a}_{\mu\nu} \equiv +b_{\mu\nu} - (\Lambda \times g_{\mu\nu})$. The elegance of this proof however does not shield us in our hunt for a unified field theory from the fact that there is no guarantee that the relationship $c_{\mu\nu} \equiv \underline{a}_{\mu\nu}$ is generally valid. Further evidence in the positive or in the negative is necessary on this issue.*

Theorem 3.2 (The tensor unknown$_{\mu\nu}$ I)**.** *A very well-founded question leads us to the problem of a common or joint tensor between the tensors* $\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})$ *and the tensor* $G_{\mu\nu}$*. Does there exist such a common tensor at all?*
Claim.

If *the premise*

$$\underbrace{+1 = +1}_{(Premise)} \tag{41}$$

is true, ***then*** *the conclusion*

$$unknown_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) - d_{\mu\nu} \tag{42}$$

is also true, the absence of any technical errors presupposed.

Proof by modus ponens. The premise

$$(+1) = (+1) \tag{43}$$

of this theorem is true. Multiplying the premise of this theorem by $\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})$ it is

$$1 \times \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \equiv 1 \times \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{44}$$

or

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{45}$$

We rearrange equation 45 as

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) - d_{\mu\nu} + d_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) + 0 \tag{46}$$

According to our definition of an anti-tensor (definition 2.1) we define the following: $unknown_{\mu\nu} \equiv \underline{d}_{\mu\nu} \equiv \left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})\right) - d_{\mu\nu}$. In this context, we decompose the equation 46 as

$$unknown_{\mu\nu} + d_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{47}$$

In general, under these conditions, the tensor unknown$_{\mu\nu}$ is determined as

$$unknown_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) - d_{\mu\nu} \tag{48}$$

□

Theorem 3.3 (The tensor unknown$_{\mu\nu}$ II)**.** *It is expected that the same tensor unknown$_{\mu\nu}$ is in the same respect a determining part of Einstein's tenosr $G_{\mu\nu}$.*
Claim.

If *the premise*

$$\underbrace{+1 = +1}_{(Premise)} \tag{49}$$

is true, ***then*** *the conclusion*

$$unknown_{\mu\nu} \equiv b_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{50}$$

is also true, the absence of any technical errors presupposed.

Proof by modus ponens. The premise

$$(+1) = (+1) \tag{51}$$

of this theorem is true. Multiplying the premise of this theorem by $(G_{\mu\nu})$ it is

$$1 \times (G_{\mu\nu}) \equiv 1 \times (G_{\mu\nu}) \tag{52}$$

or

$$G_{\mu\nu} \equiv G_{\mu\nu} \tag{53}$$

Theoretically, it is possible that Einstein's tensor $G_{\mu\nu}$ is determined by other, often simpler tensors. In our understanding, it is at least the tensor of ordinary matter, $a_{\mu\nu}$, which is a determining part of Einstein's tensor. Of course, such an approach to Einstein's tensor $G_{\mu\nu}$ does not exclude the possibility that there are circumstances where $a_{\mu\nu}$ = $G_{\mu\nu}$. However, such a condition need not to be given in general. Furthermore, as proofed before, the tensor $a_{\mu\nu}$ is a determining part of the tensor $G_{\mu\nu}$. Therefore, we need to consider that

$$G_{\mu\nu} - a_{\mu\nu} + a_{\mu\nu} \equiv G_{\mu\nu} + 0 \tag{54}$$

In this context, we define $unknown_{\mu\nu} \equiv G_{\mu\nu} - a_{\mu\nu}$. Equation 54 simplifies as

$$+ a_{\mu\nu} + unknown_{\mu\nu} \equiv G_{\mu\nu} \tag{55}$$

Rearranging equation, it is

$$+ unknown_{\mu\nu} + a_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \tag{56}$$

According to definition 2.4, equation 56 simplifies as

$$+ unknown_{\mu\nu} + a_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \equiv +a_{\mu\nu} + b_{\mu\nu} \tag{57}$$

or as

$$+ unknown_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \equiv +b_{\mu\nu} \tag{58}$$

and at the end as

$$+ unknown_{\mu\nu} \equiv +b_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{59}$$

□

Theorem 3.4 ($\text{unknown}_{\mu\nu} \equiv \text{unknown}_{\mu\nu}$)**.** ***Grain by grain and the hen fills her belly***. *In other words, advances step by step seems to be at least at this point in time the best way to approach more and more to the core of the problem of the unified field theory. Theorem 3.2 and theorem 3.3 were able to provide clear evidence and worked it out in great detail, that* ***there is a common tensor, denoted as: unknown***$_{\mu\nu}$ *and until now still of not identified structure, which determines to the same extent both, the Einstein's tensor* $G_{\mu\nu}$ *and the tensor* $\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})$*. In the following, it is valuable to check carefully the consequences of such an astonishing insight.*
Claim.

If *the premise*

$$\underbrace{+1 = +1}_{(Premise)} \tag{60}$$

is true, ***then*** *the conclusion*

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \equiv +b_{\mu\nu} + d_{\mu\nu} \tag{61}$$

is also true, the absence of any technical errors presupposed.

Proof by modus ponens. The premise

$$(+1) = (+1) \tag{62}$$

of this theorem is true. Multiplying the premise of this theorem by $\text{unknown}_{\mu\nu}$ it is

$$1 \times unknown_{\mu\nu} \equiv 1 \times unknown_{\mu\nu} \tag{63}$$

or

$$unknown_{\mu\nu} \equiv unknown_{\mu\nu} \tag{64}$$

Equation 64 is rearranged according to theorem 3.2 as

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) - d_{\mu\nu} \equiv unknown_{\mu\nu} \tag{65}$$

According to the theorem 3.3, equation 65 simplifies as

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) - d_{\mu\nu} \equiv +b_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{66}$$

Equation 66 simplifies as

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - d_{\mu\nu} \equiv +b_{\mu\nu} \tag{67}$$

In general, under the conditions above, it is

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \equiv +b_{\mu\nu} + d_{\mu\nu} \tag{68}$$

□

Theorem 3.5 (Einstein's tensor $G_{\mu\nu} \equiv a_{\mu\nu} + c_{\mu\nu}$)**.** *In particular, since gravitational waves were captured[1], it appears to be that general relativity theory is more and more on the way to become one of the icons of modern science. Therefore and in connection with other aspects, under conditions where the tensor $G_{\mu\nu}$ and the tensors $\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})$ are determined by at least by one common tensor, denoted as unknown$_{\mu\nu}$, the consequences of theorem 3.4 should be subject to further scrutiny.*
Claim.

***If** the premise*

$$\underbrace{+1 = +1}_{(Premise)} \tag{69}$$

*is true, **then** the conclusion*

$$G_{\mu\nu} \equiv +a_{\mu\nu} + c_{\mu\nu} \tag{70}$$

is also true, the absence of any technical errors presupposed.

Proof by modus ponens. The premise

$$(+1) = (+1) \tag{71}$$

of this theorem is true. Multiplying the premise of this theorem by $(G_{\mu\nu})$ it is

$$1 \times (G_{\mu\nu}) \equiv 1 \times (G_{\mu\nu}) \tag{72}$$

or

$$G_{\mu\nu} \equiv G_{\mu\nu} \tag{73}$$

Einstein's tensor $G_{\mu\nu}$ is determined as

$$G_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \tag{74}$$

This equation simplifies as (definition 2.2)

$$G_{\mu\nu} \equiv (+a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) \tag{75}$$

and equally as

$$G_{\mu\nu} \equiv (+a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}) - (+b_{\mu\nu} + d_{\mu\nu}) \tag{76}$$

or as

$$G_{\mu\nu} \equiv +a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} - b_{\mu\nu} - d_{\mu\nu} \tag{77}$$

In general, it is

$$G_{\mu\nu} \equiv +a_{\mu\nu} + c_{\mu\nu} \tag{78}$$

□

Remark 3.2. *Theorem 3.5 has provided striking evidence that Einstein's tensor $G_{\mu\nu}$ can be decomposed or is determined in general as $G_{\mu\nu} \equiv +a_{\mu\nu} + c_{\mu\nu}$. However, even if no sweeping generalisation can be made that this theorem (theorem 3.5) is generally valid, this theorem would be of little or of none worth if there were no circumstances where the condition $\left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \equiv +b_{\mu\nu} + d_{\mu\nu}\right)$ is valid. Because of these and similar objections, the phrase is always right that such a regrettable generalisation could be wrong as long as it is not possible to verify the result of this theorem (equation 78) through other theoretical or experimental investigations. However and in deductivist style, under conditions where $\left(\frac{R}{2}\right) \times g_{\mu\nu} \equiv +b_{\mu\nu} + d_{\mu\nu}$ (see theorem 3.4, equation 68) the inferences are valid and it is $G_{\mu\nu} \equiv +a_{\mu\nu} + c_{\mu\nu}$. However, future research is welcomed to provide additional evidence on this topic.*

Theorem 3.6 (The field $c_{\mu\nu}$ + $d_{\mu\nu}$). ***Claim.***

If *the premise*

$$\underbrace{+1 = +1}_{(Premise)} \tag{79}$$

is true, ***then*** *the conclusion*

$$c_{\mu\nu} + d_{\mu\nu} \equiv \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) - (\Lambda \times g_{\mu\nu}) \tag{80}$$

is also true, the absence of any technical errors presupposed.

Proof by modus ponens. The premise

$$(+1) = (+1) \tag{81}$$

of this theorem is true. Multiplying the premise of this theorem by the Ricci tensor $R_{\mu\nu}$ it is

$$1 \times R_{\mu\nu} \equiv 1 \times R_{\mu\nu} \tag{82}$$

or

$$R_{\mu\nu} \equiv R_{\mu\nu} \tag{83}$$

According to definition 2.2, this equation is rearranged as

$$a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} \equiv R_{\mu\nu} \tag{84}$$

The sum of the tensors $c_{\mu\nu}$ + $d_{\mu\nu}$ follows as

$$c_{\mu\nu} + d_{\mu\nu} \equiv R_{\mu\nu} - a_{\mu\nu} - b_{\mu\nu} \tag{85}$$

or as

$$c_{\mu\nu} + d_{\mu\nu} \equiv R_{\mu\nu} - (a_{\mu\nu} + b_{\mu\nu}) \tag{86}$$

or according to definition 2.4 as

$$c_{\mu\nu} + d_{\mu\nu} \equiv R_{\mu\nu} - \left(\frac{2 \times \pi \times \gamma \times T}{c^4} \right) \times g_{\mu\nu} \tag{87}$$

Based on Einstein's field equations(definition 2.6), this equation is equivalent with

$$c_{\mu\nu} + d_{\mu\nu} \equiv \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) - (\Lambda \times g_{\mu\nu}) \tag{88}$$

□

Theorem 3.7 (The tensor unknown$_{\mu\nu}$ equals c$_{\mu\nu}$)**.** *The tensor unknown$_{\mu\nu}$ is identical with the tensor of time/gravitational field[2] $c_{\mu\nu}$ as associated with the tensor of ordinary matter $a_{\mu\nu}$.*
Claim.

***If** the premise*

$$\underbrace{+1 = +1}_{(Premise)} \tag{89}$$

*is true, **then** the conclusion*

$$unknown_{\mu\nu} \equiv +c_{\mu\nu} \tag{90}$$

is also true, the absence of any technical errors presupposed.

Proof by modus ponens. The premise

$$(+1) = (+1) \tag{91}$$

of this theorem is true. Multiplying the premise of this theorem by $\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})$ it is

$$1 \times \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \equiv 1 \times \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{92}$$

or

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{93}$$

We rearrange equation 93 as

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) - d_{\mu\nu} + d_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) + 0 \tag{94}$$

According to our definition of an anti-tensor (definition 2.1) we define the following: $unknown_{\mu\nu} \equiv \underline{d}_{\mu\nu} \equiv \left(\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})\right) - d_{\mu\nu}$. In this context, we rearrange equation 46 as

$$unknown_{\mu\nu} + d_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \tag{95}$$

Equation 95 changes according to theorem 3.6 as

$$unknown_{\mu\nu} + d_{\mu\nu} \equiv +c_{\mu\nu} + d_{\mu\nu} \tag{96}$$

Simplifying equation 96, we obtain

$$unknown_{\mu\nu} \equiv +c_{\mu\nu} \tag{97}$$

□

Theorem 3.8 (The unity of gravitation and electromagnetism $b_{\mu\nu}$ + $c_{\mu\nu}$). *What is gravitation? What is electromagnetism? Do they exist independently of each other and the geometry of space-time? Historically, the trial to describe all fundamental interactions in the sense of a unified field theory by objects of space-time geometry is ascribed especially to Hermann Klaus Hugo Weyl (1885-1955) [44], [45] and Sir Arthur Stanley Eddington (1882-1944) [13], [14]. Weyl's generalisation of Riemannian geometry and his classical geometrical approach connected the gravitational and the electromagnetic fields as represented by the metrical field and a vector field. Eddington tried to provide physics with new foundations that covered everything in the physical world, from the universe at large to the tiny particle. However, one of the grand failures of Eddington's philosophy of physics was that the laws of physics were treated as constructions[14] of the physicists and were not expressions of processes or regularities in an external, objective and real world. Kaluza's[30] introduced five dimensions in order to approach to the unification of gravitation and electromagnetism. In point of fact and from an epistemological standpoint, neither Einstein nor the majority of other physicist were impressed by these and similar contributions.*

Claim.

If *the premise*

$$\underbrace{+1 = +1}_{(Premise)} \tag{98}$$

is true, ***then*** *the conclusion*

$$b_{\mu\nu} + c_{\mu\nu} \equiv \left(\left(\frac{1}{8 \times \pi} \times \left((F_{\mu\, c} \times F^{\mu\, c}) - (F_{de} \times F^{de})\right)\right) - \Lambda\right) \times g_{\mu\nu} \tag{99}$$

is also true, the absence of any technical errors presupposed.

Proof. The premise

$$(+1) = (+1) \tag{100}$$

of this theorem is true. Multiplying this premise by $\left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right)$, the stress-energy tensor of matter, it is

$$1 \times \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \equiv 1 \times \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \tag{101}$$

or

$$\left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \tag{102}$$

Equation 102 changes according to equation 11 (definition 2.4)

$$(G_{\mu\nu} + (\Lambda \times g_{\mu\nu})) \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \tag{103}$$

According to equation 78 (theorem 3.5) it is $+a_{\mu\nu} + c_{\mu\nu} \equiv G_{\mu\nu}$. Equation 103 can be rearranged as

$$(+a_{\mu\nu} + c_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \tag{104}$$

Equation 11 (definition 2.4) is $+a_{\mu\nu} + b_{\mu\nu} \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right)$. Therefore, equation 104 can be rearranged as

$$(+a_{\mu\nu} + c_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) \equiv (+a_{\mu\nu} + b_{\mu\nu}) \tag{105}$$

Equation 105 can be simplified further as

$$(+c_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) \equiv (+b_{\mu\nu}) \tag{106}$$

In general, we must accept that the stress-energy tensor of the electromagnetic field (denoted as $b_{\mu\nu}$) is determined by the relationship $+b_{\mu\nu} \equiv (+c_{\mu\nu}) + (\Lambda \times g_{\mu\nu})$ while the tensor of time/gravitational field as associated with the tensor of ordinary matter (denoted by $a_{\mu\nu}$) is denoted by the tensor $c_{\mu\nu}$. In the same context, the tensor of time/gravitational field ($c_{\mu\nu}$) is determined among other by the stress-energy tensor of the electromagnetic field ($b_{\mu\nu}$) as

$$(+c_{\mu\nu}) \equiv (+b_{\mu\nu}) - (\Lambda \times g_{\mu\nu}) \tag{107}$$

It is $b_{\mu\nu} \equiv \frac{1}{4 \times 4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \times g_{\mu\nu}$ as demonstrated by equation 20 (definition 2.11). Equation 107 becomes more concrete as

$$(+c_{\mu\nu}) \equiv \left(\frac{1}{4 \times 4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \times g_{\mu\nu} \right) - (\Lambda \times g_{\mu\nu}) \tag{108}$$

or as

$$c_{\mu\nu} \equiv \left(\left(\frac{1}{16 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \right) - \Lambda \right) \times g_{\mu\nu} \tag{109}$$

Equation 107 demands that $c_{\mu\nu} \equiv b_{\mu\nu} - (\Lambda \times g_{\mu\nu})$ Adding the tensor $\mathrm{b}_{\mu\nu}$ to equation 107 yields the tensor of gravitation and electromagnetism as

$$b_{\mu\nu} + c_{\mu\nu} \equiv b_{\mu\nu} + b_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{110}$$

or in more detail

$$b_{\mu\nu} + c_{\mu\nu} \equiv 2 \times \left(\frac{1}{4 \times 4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \times g_{\mu\nu} \right) - (\Lambda \times g_{\mu\nu}) \tag{111}$$

or

$$b_{\mu\nu} + c_{\mu\nu} \equiv \left(\frac{1}{8 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \times g_{\mu\nu} \right) - (\Lambda \times g_{\mu\nu}) \tag{112}$$

and finally

$$b_{\mu\nu} + c_{\mu\nu} \equiv \left(\left(\frac{1}{8 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \right) - \Lambda \right) \times g_{\mu\nu} \tag{113}$$

□

Remark 3.3. *It is the successful geometrization[9] of Einstein's field equations which is the foundation of "**. . . joining the gravitational and the electromagnetic field into one single hyperfield whose equations represent the conditions imposed on the geometrical structure . . .** "[see 42, p. 5]*

IV. Discussion

Historically, there have been many different attempts [5], [6] to unify Maxwell's theory of the electromagnetic[34] field with Einstein's general theory of gravitation[16], [19]–[21] into a unified field theory[17]. A geometrization of the stress-energy tensor of the electromagnetic field and the stress-energy tensor of matter of Einstein's field equations in particular has been an indispensable foundation for the development of **a completely geometrized[25] theory of relativity** in order to overcome the difficulties inherent in this matter. Advantage is taken of a previous gemetriziation[9] of Einstein's field equation. A generally covariant field equation for gravitation and electromagnetism is developed as $b_{\mu\nu} + c_{\mu\nu} \equiv \left(\left(\frac{1}{8 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \right) - \Lambda \right) \times g_{\mu\nu}$ by considering especially the relationship to the metric tensor $\mathrm{g}_{\mu\nu}$. In order to formulate a logically consistent theory of quantum gravity or an unified field theory, it is necessary to change drastically several points of view. Equation 107 of theorem 3.8 is demanding that the stress-energy tensor of the electromagnetic field (denoted by $\mathrm{b}_{\mu\nu}$) is a determining part of he tensor of time/gravitational field denoted by the tensor $\mathrm{c}_{\mu\nu}$ as $+c_{\mu\nu} \equiv +b_{\mu\nu} - (\Lambda \times g_{\mu\nu})$. In this context, equation 107 or equation 113 justifies the question, does the gravitational field ($\mathrm{c}_{\mu\nu}$) itself possess any kind of an energy density, stress et cetera or not? The question is complicated more especially since Einstein himself introduced a mathematical object in his final general theory of relativity for describing the energy components of the gravitational field by $\mathrm{t}_{\sigma}{}^{\alpha}$. "**Es ist zu beachten dass $\mathrm{t}_{\sigma}{}^{\alpha}$ kein Tensor ist; . . . Die Größen $\mathrm{t}_{\sigma}{}^{\alpha}$ bezeichnen wir als die 'Energiekomponenten'des Gravitationsfelds.** "[see 16, p. 806] However, soon Schrödinger[39] and Bauer[10] showed that the energy components of the gravitational field are **equal to zero** when the same should not be, and are **not equal to zero** when the same should be. The problem of the energy of the gravity field in general relativity appears to be solved or is at least solvable by equation 113. It is to be noted, that **per definitionem** all energy, radiation, stress et cetera is included exclusively within the stress-energy tensor of matter, denoted by $\mathrm{E}_{\mu\nu} = \mathrm{a}_{\mu\nu} + \mathrm{b}_{\mu\nu}$. Thus far, there is and cannot be any further energy left, which can be treated as being part of the gravitational field $\mathrm{c}_{\mu\nu}$ itself. However, this does not exclude that time/gravitational field $\mathrm{c}_{\mu\nu}$ is determined by the stress-energy tensor of the electromagnetic field (denoted by $\mathrm{b}_{\mu\nu}$). It is necessary to make a distinction between being determined by energy, radiation, stress et cetera and to posses itself energy, radiation, stress et cetera. Therefore, **the energy components of the gravitational field** [10], [22], [39] of Einstein's general theory of relativity and require a further and definite clarification. A question justified is, are there any circumstances possible where

the energy components of the gravitational field [10], [22], [39] are more or less identical with the stress-energy tensor of the electromagnetic field (denoted by $b_{\mu\nu}$). In other words, is it possible to find conditions where the stress-energy tensor of the electromagnetic field and **the energy components of the gravitational field** are related something similar with $t_{\sigma}{}^{\alpha} \approx \left(\left(\frac{1}{8 \times \pi} \times \left(\left(F_{\mu\, c} \times F^{\mu\, c} \right) - \left(F_{de} \times F^{de} \right) \right) \right) - \Lambda \right)$.

V. Conclusion

If was possible to couple the stress-energy tensor of electromagnetic field ($b_{\mu\nu}$) and the tensor of time/gravitational field ($c_{\mu\nu}$) to the metric tensor ($g_{\mu\nu}$)itself and to unify both fields into a hyper-field of gravitation and electromagnetism.

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