The exact value of the cosmological constant Λ

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Abstract* — *Aim: The theoretical value of the cosmological constant Λ and the problem associated with the same is reviewed again. Methods: The stress-energy-tensor was geometrized.
Results: Based on the geometrized stress-energy tensor, it was possible to calculate the exact value the cosmological constant.
Conclusion: The theoretical value of the cosmological constant Λ can be calculated very precisely.

Keywords — ***Cosmological constant, Gravitation, Electromagnetism, Einstein's field equations, Unified field theory.***

I. Introduction

Energy, time (Barukčić, 2011) and space are deeply interrelated. Especially gravity as the dominant interaction at large length scales is an essential part of cosmology. Einstein (Einstein, 1916; Einstein & de Sitter, 1932; Einstein, 1915, 1917) introduced a new way of representing gravity by replacing the single gravitational potential and the associated field equation of Newton's theory. One of the basic features of Einstein's theory of general relativity (GTR) and equally that what distinguishes GTR sharply from all other competing physical theories, is the geometrization of a physical interaction which opened the theoretical possibility to understand the gravitational field as something like the manifestation of space - time curvature.Einstein's point of view was that the gravitational field can be described by using particular mathematical tools like a metric tensor $g_{\mu\nu}$. However, this need not imply that gravity is and has to be reduced to geometry in its own right. In point of fact, Einstein's stress-energy momentum tensor of GTR is a weak spot of his theory because this field is thus far devoid (Goenner, 2004) of any geometrical significance. Various proposals for a unified field theory "**a generalization of the theory of the gravitational field**"(Einstein, 1950) were influenced by the desired replacement of the stress-energy momentum tensor of matter by geometrical structures. In order to bring some order into the many different ways to include the electromagnetic field into a geometric setting, general relativity (Barukčić, 2016a) can serve as a point of departure for this undertaking. However, I do not see any reason to assume that 'geometrization " and 'unification 'are incompatible. Still, both need not to be conceptually identical. A complete geometrization of Einstein's gravitational field equations could eventually end up at a unified field theory in the sense of Weyl and Eddington's classical field theory in which all fundamental interactions are described by objects of space-time geometry. Besides of such fundamental problems, other and much more simple problems are not solved too. What is the value of the cosmological constant Λ (Einstein & de Sitter, 1932; Einstein, 1917) ? Trying to answer these and similar questions was the subject of many publications and is of this paper too.

II. Material and Methods

The Royal Society of London and the Royal Astronomical Society announced at their joint meeting on the sixth of November 1919 that astronomical observations made by a special British team during the solar eclipse on May 29 provided the first empirical test of the validity of Einstein's general theory of relativity. In order to obtain a kind of a deeper knowledge of the foundations of nature and physics as such it seems therefore that the basic concepts should be in accordance with Einstein's general of relativity (Einstein, 1916) from the beginning. In point of fact, attempts to extend general relativity's geometrization of gravitational force to non-gravitational interactions, in particular, to electromagnetism (Barukčić, 2016a), were not in vain.

Definitions

Definition 3.1 (Anti tensor). *Let $a_{\mu\nu}$ denote a co-variant (lower index) second-rank tensor. Let $b_{\mu\nu}$ denote another co-variant second-rank et cetera. Let $E_{\mu\nu}$ denote the sum of these co-variant second-rank tensors. Let the relationship $a_{\mu\nu} + b_{\mu\nu} + ... \equiv E_{\mu\nu}$ be given. A co-variant second-rank anti tensor (Barukčić, 2020) of a tensor $a_{\mu\nu}$ denoted in general as $\underline{a}_{\mu\nu}$ is defined*

$$\begin{aligned} \underline{a}_{\mu\nu} &\equiv E_{\mu\nu} - a_{\mu\nu} \\ &\equiv b_{\mu\nu} + ... \end{aligned} \tag{1}$$

Let $a^{\mu\nu}$ denote a contra-variant (upper index) second-rank tensor. Let $b^{\mu\nu}$ denote another contra-variant (upper index) second-rank et cetera. Let $E^{\mu\nu}$ denote the sum of these contra-variant (upper index) second-rank tensors. Let the relationship $a^{\mu\nu} + b^{\mu\nu} + ... \equiv E^{\mu\nu}$ be given. A co-variant second-rank anti tensor of a tensor $a^{\mu\nu}$ denoted in general as $\underline{a}^{\mu\nu}$ is defined

$$\begin{aligned} \underline{a}^{\mu\nu} &\equiv E^{\mu\nu} - a^{\mu\nu} \\ &\equiv b^{\mu\nu} + ... \end{aligned} \tag{2}$$

Let ${a_{\mu}}^{\nu}$ denote a mixed second-rank tensor. Let ${b_{\mu}}^{\nu}$ denote another mixed second-rank et cetera. Let ${E_{\mu}}^{\nu}$ denote the sum of these mixed second-rank tensors. Let the relationship ${a_{\mu}}^{\nu} + {b_{\mu}}^{\nu} + ... \equiv {E_{\mu}}^{\nu}$ be given. A mixed second-rank anti tensor of a tensor ${a_{\mu}}^{\nu}$ denoted in general as ${\underline{a}_{\mu}}^{\nu}$ is defined

$$\begin{aligned} {\underline{a}_{\mu}}^{\nu} &\equiv {E_{\mu}}^{\nu} - {a_{\mu}}^{\nu} \\ &\equiv {b_{\mu}}^{\nu} + ... \end{aligned} \tag{3}$$

Symmetric tensors of rank 2 may represent many physical properties objective reality. A co-variant second-rank tensor $a_{\mu\nu}$ is symmetric if

$$a_{\mu\nu} \equiv a_{\nu\mu} \tag{4}$$

However, there are circumstances, where a tensor is anti-symmetric. A co-variant second-rank tensor $a_{\mu\nu}$ is anti-symmetric if

$$a_{\mu\nu} \equiv -a_{\nu\mu} \tag{5}$$

Thus far, there are circumstances were an anti-tensor is identical with an anti-symmetrical tensor.

$$a_{\mu\nu} \equiv E_{\mu\nu} - b_{\mu\nu} + ... \equiv E_{\mu\nu} - \underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{6}$$

Under conditions where $E_{\mu\nu} = 0$, an anti-tensor is identical with an anti-symmetrical tensor or it is

$$-\underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{7}$$

However, an anti-tensor is not identical with an anti-symmetrical tensor as such.

Definition 3.2 (Einstein's field equations). *Let $R_{\mu\nu}$ denote the Ricci tensor (Ricci & Levi-Civita, 1900) of 'Einstein's general theory of relativity'(Einstein, 1916), a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. Let R denote the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold. Ricci scalar curvature is the contraction of the Ricci tensor and is written as R without subscripts or arguments. Let Λ denote the Einstein's cosmological constant. Let $\underline{\Lambda}$ denote the "**anti cosmological constant**"(Barukčić, 2015). Let $g_{\mu\nu}$ metric tensor of Einstein's general theory of relativity. Let $G_{\mu\nu}$ denote Einstein's curvature tensor. Let $\underline{G}_{\mu\nu}$ denote the "**anti tensor**"(Barukčić, 2016c) of Einstein's curvature tensor. Let $E_{\mu\nu}$ denote stress-energy tensor of energy. Let $\underline{E}_{\mu\nu}$ denote tensor of non-energy, the anti-tensor of the stress-energy tensor of energy. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field. Let c denote the speed of the light in vacuum, let γ denote Newton's gravitational "**constant**"(Barukčić, 2014, 2015, 2016b, 2016c). Let π denote the number pi. Einstein's field equation, published by Albert Einstein (Einstein, 1915) for the first time in 1915, and finally 1916 (Einstein, 1916) but later with the "**cosmological constant**"(Einstein, 1935; Einstein & de Sitter, 1932; Einstein, 1917) term are determined as*

$$R_{\mu\nu} - \left(\left(\frac{R}{2} \right) \times g_{\mu\nu} \right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4} \right) \times T_{\mu\nu} \equiv E_{\mu\nu} \tag{8}$$

However, the above left-hand side of the Einstein field equations represents only one part (Ricci curvature) of the geometric structure (Weyl curvature).

Definition 3.3 (**Laue's scalar T**). *Max von Laue (1879-1960) proposed the meanwhile so called Laue scalar (Laue, 1911) (criticised by Einstein (Einstein & Grossmann, 1913)) as the contraction of the the stress–energy momentum tensor $T_{\mu\nu}$ denoted as T and written without subscripts or arguments. Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, it is*

$$T \equiv g^{\mu\nu} \times T_{\mu\nu} \tag{9}$$

Taken Einstein seriously, $T_{\mu\nu}$ "denotes the co-variant energy tensor of matter"(see Einstein, 1923, p. 88). In other words, "Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense."(see Einstein, 1923, p. 93)

Definition 3.4 (**The entity E**). *In general, we define the entity E as*

$$\begin{aligned} E &\equiv \left(\frac{8 \times \pi \times \gamma}{c^4 \times D}\right) \times T \\ &\equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \end{aligned} \tag{10}$$

*where D is the space-time dimension, where c denote the speed of the light in vacuum, γ denote Newton's gravitational "**constant**"(Barukčić, 2014, 2015, 2016b, 2016c), π is the number pi and T denote Laue's scalar.*

Lemma 3.1 (**The relationship between the entity E and the dimension of space-time D**).

Einstein Field Equations in other space-time dimensions (see Málek, 2012, p. 31) than 3+1 need not lead to insurmountable contradictions.
Claim.
In general, the entity E is given by

$$E \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{11}$$

Proof By Modus Ponens.
If the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{12}$$

is true, **then** the conclusion

$$E \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{13}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{14}$$

is true. Multiplying this premise by the stress-energy momentum tensor it is

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{15}$$

We do expect that the stress-energy momentum tensor can be geometrized completely as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv E \times g_{\mu\nu} \tag{16}$$

Rearranging it is,

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \equiv E \times g_{\mu\nu} \times g^{\mu\nu} \tag{17}$$

According to definition of Laue's scalar (definition 3.3) it is

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T \equiv E \times g_{\mu\nu} \times g^{\mu\nu} \tag{18}$$

According to definition 3.14 (definition 3.14, equation 43) it is

$$\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \equiv E \times D \tag{19}$$

The entity E is depending on the number of space-time dimensions D and follows as

$$E \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{20}$$

In other words, our conclusion is true.
Quod erat demonstrandum.

Definition 3.5 (**The tensor of energy and momentum a$_{\mu\nu}$ + b$_{\mu\nu}$**). *The tensor of stress-energy-momentum denoted as $E_{\mu\nu}$ is determined in detail as follows.*

$$\begin{aligned}
E_{\mu\nu} &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\
&\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \\
&\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \\
&\equiv R_{\mu\nu} - \underline{E}_{\mu\nu} \\
&\equiv a_{\mu\nu} + b_{\mu\nu} \\
&\equiv E \times g_{\mu\nu}
\end{aligned} \tag{21}$$

In our understanding, the stress-energy tensor of the electromagnetic field (b$_{\mu\nu}$) is equivalent to the portion of the stress-energy tensor of matter / energy (E$_{\mu\nu}$)due to the electromagnetic field where where T$_{\mu\nu}$ "denotes the co-variant energy tensor of matter"(see Einstein, 1923, p. 88). In other words, there is no third tensor between the stress-energy tensor of the electromagnetic field (b$_{\mu\nu}$) and the tensor of ordinary matter or matter in the narrower sense (a$_{\mu\nu}$), **a third tensor is not given, tertium non datur**! In other words, as outlined view lines before: "*Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense.* "(see Einstein, 1923, p. 93)

Electromagnetic field $b_{\mu\nu}$

Ordinary matter a$_{\mu\nu}$

Figure. Energy tensor as identity of ordinary matter and electromagnetic field.

Vranceanu (see Vranceanu, 1936) is elaborating on the same issue too. In point of fact, the energy tensor T$_{kl}$ is treated by Vranceanu as the sum of two tensors one of which is due to the electromagnetic field (b$_{\mu\nu}$).

> "On peut aussi supposer que le tenseur d'énergie T$_{kl}$ soit la somme de deux tenseurs dont un dû au champ électromagnétique . . . "(see Vranceanu, 1936)

Translated into English: '*One can also assume that the energy tensor T_{kl} be the sum of two tensors one of which is due to the electromagnetic field.*'In this context, it is necessary to make a distinction between the relationship between ordinary matter and electromagnetic field and matter and gravitational field. Matter and ordinary matter are not completely the same.

Definition 3.6 (**The tensor of non-energy**). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor of non-energy or the anti tensor of the stress energy tensor is defined/derived/determined as follows:*

$$\underline{E}_{\mu\nu} \equiv R_{\mu\nu} - \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) \equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}\right) \equiv c_{\mu\nu} + d_{\mu\nu} \tag{22}$$

Definition 3.7 (The anti Einstein's curvature tensor or the tensor or non-curvature). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor of non-curvature is defined/derived/determined as follows:*

$$\underline{G}_{\mu\nu} \equiv R_{\mu\nu} - G_{\mu\nu} \equiv R_{\mu\nu} - \left(R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right)\right) \equiv \left(\frac{R}{2}\right) \times g_{\mu\nu} \equiv b_{\mu\nu} + d_{\mu\nu} \tag{23}$$

Definition 3.8 (The tensor $d_{\mu\nu}$ (neither curvature nor momentum)). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor of neither curvature nor momentum is defined/derived/determined as follows:*

$$d_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - b_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu}) - c_{\mu\nu} \tag{24}$$

There may exist circumstances where this tensor indicates pure vacuum, the space devoid of any matter.

Definition 3.9 (The tensor $c_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor of non-momentum and curvature is defined/derived/determined as follows:*

$$c_{\mu\nu} \equiv b_{\mu\nu} - (\Lambda \times g_{\mu\nu}) \tag{25}$$

Definition 3.10 (The tensor $b_{\mu\nu}$). *The co-variant stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, is of order two and its components can be displayed by a 4 × 4 matrix too. Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor $b_{\mu\nu}$ denotes the stress-energy tensor of the electromagnetic field (Hughston & Tod, 1990, p. 38) expressed more compactly and in a coordinate-independent is*

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{26}$$

where F_{de} is called the (traceless) Faraday/electromagnetic/field strength tensor.

Definition 3.11 (The stress-energy tensor of ordinary matter $a_{\mu\nu}$). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the stress-energy tensor of ordinary matter $a_{\mu\nu}$ is defined/derived/determined as follows:*

$$a_{\mu\nu} \equiv \left(\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}\right) - b_{\mu\nu} \equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) - b_{\mu\nu} \equiv R_{\mu\nu} - (R \times g_{\mu\nu}) + (\Lambda \times g_{\mu\nu}) + d_{\mu\nu} \tag{27}$$

or

$$a_{\mu\nu} \equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) - \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\, c} \times F_{\nu\, d} \times g^{cd}\right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{de} \times F^{de}\right)\right)\right) \tag{28}$$

Definition 3.12 (The Ricci tensor $R_{\mu\nu}$). *Let $R_{\mu\nu}$ denote the Ricci tensor (Ricci & Levi-Civita, 1900) of 'Einstein's general theory of relativity'(Einstein, 1916), a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy* *tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field.*

$$
\begin{aligned}
R_{\mu\nu} &\equiv \underbrace{\left(\left(\frac{4\times 2\times \pi \times \gamma}{c^4}\right)\times T_{\mu\nu}\right)}_{a_{\mu\nu}+b_{\mu\nu}} + \underbrace{\left(\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) - (\Lambda \times g_{\mu\nu})\right)}_{c_{\mu\nu}+d_{\mu\nu}} \\
&\equiv (a_{\mu\nu}+b_{\mu\nu})+(c_{\mu\nu}+d_{\mu\nu}) \\
&\equiv (a_{\mu\nu}+c_{\mu\nu})+(b_{\mu\nu}+d_{\mu\nu}) \\
&\equiv (a_{\mu\nu})+(+b_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu}) \\
&\equiv (b_{\mu\nu})+(+a_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu}) \\
&\equiv (c_{\mu\nu})+(+a_{\mu\nu}+b_{\mu\nu}+d_{\mu\nu}) \\
&\equiv (d_{\mu\nu})+(+a_{\mu\nu}+b_{\mu\nu}+c_{\mu\nu}) \\
&\equiv a_{\mu\nu}+b_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu} \\
&\equiv S\times g_{\mu\nu} \\
&\equiv \left(\frac{R}{D}\right)\times g_{\mu\nu}
\end{aligned} \tag{29}
$$

Lemma 3.2 (**The relationship between the entity S and the dimension of space-time D**).

Einstein Field Equations are defined in space-time dimensions (see Málek, 2012, p. 31) other than 3+1 too.
Claim.
In general, the entity S is given by

$$S \equiv \left(\frac{R}{D}\right) \tag{30}$$

Proof By Modus Ponens.
If the premise

$$\underbrace{+1=+1}_{(Premise)} \tag{31}$$

is true, **then** the conclusion

$$S \equiv \left(\frac{R}{D}\right) \tag{32}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1)=(+1) \tag{33}$$

is true. Multiplying this premise by the stress-energy momentum tensor it is

$$R_{\mu\nu} \equiv R_{\mu\nu} \tag{34}$$

We do expect that the Ricci tensor is completely determined by the entitity S and the metric tensor $g_{\mu\nu}$ as

$$R_{\mu\nu} \equiv S\times g_{\mu\nu} \tag{35}$$

Rearranging it is,

$$R_{\mu\nu}\times g^{\mu\nu} \equiv S\times g_{\mu\nu}\times g^{\mu\nu} \tag{36}$$

or in accordance to definition 3.13

$$R \equiv S\times g_{\mu\nu}\times g^{\mu\nu} \tag{37}$$

According to definition 3.14 (definition 3.14, equation 43) it is

$$R \equiv S\times D \tag{38}$$

The entity S is depending on the number of space-time dimensions D and follows as

$$S \equiv \left(\frac{R}{D}\right) \tag{39}$$

In other words, our conclusion is true.
Quod erat demonstrandum.

Remark 3.1. *The complete geometrization of Einstein field equations as provided by Ilija Barukčić (see Barukčić, 2020) has been derived under conditions where the number of space-time dimensions D is equal to D = 4.*

Definition 3.13 (The Ricci scalar R). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric is determined at each point in space-time by lamda Λ and anti-lamda (Barukčić, 2015) $\underline{\Lambda}$ as*

$$R \equiv g^{\mu\nu} \times R_{\mu\nu} \equiv (\Lambda) + (\underline{\Lambda}) \equiv D \times S \tag{40}$$

where D is the number of space-time dimension and $S \equiv \left(\frac{R}{D}\right)$ (lemma 3.2, equation 39). A Ricci scalar curvature R which is positive at a certain point indicates that the volume of a small ball about the point has smaller volume than a ball of the same radius in Euclidean space. In contrast to this, a Ricci scalar curvature R which is negative at a certain point indicates that the volume of a small ball is larger than it would be in Euclidean space. In general it is

$$R \times g_{\mu\nu} \equiv (\Lambda \times g_{\mu\nu}) + (\underline{\Lambda} \times g_{\mu\nu}) \tag{41}$$

The cosmological constant can also be written algebraically as part of the stress–energy tensor, a second order tensor as the source of gravity (energy density).

Table 1 provides an overview of the definitions of the four basic (Barukčić, 2016a, 2016c) fields of nature.

		Curvature		
		YES	NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Tabelle 1: Einstein field equations and the four basic fields of nature

Definition 3.14 (The inverse metric tensor $g^{\mu\nu}$ and the metric tensor $g_{\mu\nu}$). *Einstein field equations relate (local) space-time curvature with (local) energy and momentum as*

$$\underbrace{R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu})}_{(local)\ space-time\ curvature} \equiv \underbrace{\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}}_{(local)\ energy\ and\ momentum} \tag{42}$$

The expression on the left side of Einstein field equations represents the curvature of space-time as determined by the metric while the expression on the right side of Einstein field equations represents the matter–energy content of space-time. Mathematically, it is possible to take the trace with respect to the metric of both sides of the Einstein field equations and it is necessary to consider circumstances that

$$g_{\mu\nu} \times g^{\mu\nu} \equiv D \tag{43}$$

where D is the number of space-time dimensions. But nonetheless, Einstein field equations (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) were initially formulated by Einstein himself in the context of a four-dimensional theory even though Einstein field equations need not to break down under conditions of D space-time dimensions (see Stephani, 2003). Therefore, based on Einstein's statement (Einstein, 1916, p. 796), one gets

$$g_{\mu\nu} \times g^{\mu\nu} \equiv D \equiv +4 \tag{44}$$

or

$$\frac{1}{g_{\mu\nu} \times g^{\mu\nu}} \equiv \frac{1}{4} \tag{45}$$

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other.

Einstein's point of view is that "... in the general theory of relativity ... must be ... the tensor $g_{\mu\nu}$ of the gravitational potential"(Einstein, 1923, p. 88) Treat the metric tensor $g_{\mu\nu}$ as a square matrix. The inverse metric tensor $g^{\mu\nu}$ is of the same size. Thus far, whatever $g_{\mu\nu}$ does, $g^{\mu\nu}$ undoes and their product is the identity.

Definition 3.15 (**Index raising**). *For an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices (Kay, 1988) raises each index. In simple words, it is*

$$F^{\left(\begin{smallmatrix}1 & 3\\ \mu & c\end{smallmatrix}\right)} \equiv g^{\left(\begin{smallmatrix}1 & 2\\ \mu & \nu\end{smallmatrix}\right)} \times g^{\left(\begin{smallmatrix}3 & 4\\ c & d\end{smallmatrix}\right)} \times F_{\left(\begin{smallmatrix}\nu & d\\ 2 & 4\end{smallmatrix}\right)} \tag{46}$$

or more professionally

$$F^{\mu\, c} \equiv g^{\mu\nu} \times g^{cd} \times F_{\nu\, d} \tag{47}$$

3.1 Axioms

3.2 Axioms in general

Axioms (Hilbert, 1917) and rules which are chosen carefully can be of use to avoid logical inconsistency and equally preventing science from supporting particular ideologies. Rightly or wrongly, long lasting advances in our knowledge of nature are enabled by suitable axioms (Easwaran, 2008) too. Einstein himself brings it again to the point. (see Einstein, 1919, p. 17)

> "**Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden**."(see Einstein, 1919, p. 17)

Einstein's previous position now been translated into English: *The truly great advances in our understanding of nature originated in a manner almost diametrically opposed to induction.* It is worth mentioning in this matter, Einstein himself advocated especially basic laws (axioms) and conclusions derived from the same as a main logical foundation of any 'theory'.

> "**Grundgesetz (Axiome)**
> und
> **Folgerungen**
> zusammen bilden das was man
> **eine 'Theorie'**
> nennt. "(see Einstein, 1919, p. 17)

Albert Einstein's (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a 'theory'* has still to get round. However, it is currently difficult to ignore completely these historical and far reaching words of wisdom. The same taken more seriously and put into practice, will yield an approach to fundamental scientific problems which is more creative and sustainably logically consistent. Historically, Aristotle himself already cited **the law of excluded middle** and **the law of contradiction** as examples of axioms. However, **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally as the most simple axiom of science. Something which is really just itself is equally different from everything else. In point of fact, is such an equivalence which everything has to itself inherent or must the same be constructed by human mind and consciousness. Following Gottfried Wilhelm Leibniz (1646-1716):

> "**Chaque chose est ce qu'elle est.
> Et dans autant d'exemples qu'on voudra
> A est A, B est B**. "(see Leibniz, 1765, p. 327)

or **A = A**, **B = B** or **+1 = +1**. In this context, **lex contradictionis**, the negative of lex identitatis, or **+0 = +1** is of no minor importance too.

3.2.1 Axiom I. Lex identitatis

To say that +1 is identical to +1 is to say that both are the same.
Axiom 1. Lex identitatis.

$$+1 \equiv +1 \tag{48}$$

However, even such a numerical identity which seems in itself wholly unproblematic, for it indicates just to a relation which something has to itself and nothing else, is still subject to controversy. Another increasingly popular view is that the same numerical identity implies the controversial view that we are talking about two different numbers +1. The one +1 is on the left side on the equation, the other +1 is on the right side of an equation. The basicness of the relation of identity implies the contradiction too while circularity is avoided. In other words, how can the same +1 be identical with itself and be equally different from itself? We may usefully state that identity is an utterly problematic notion and might be the most troublesome of all.

3.2.2 Axiom II. Lex contradictionis

Axiom 2. Lex contradictionis.

$$+0 \equiv +1 \tag{49}$$

A considerable obstacle to understanding contemporary usage of the term contradiction, however, is that contradiction does not seem to be a unitary one. How can something be both, itself (**a path is a straight line** from the standpoint of a co-moving observer at a certain point in space-time) **and** the other of itself, its own opposition (**the same path is not a straight line**, the same path is curved, from the standpoint of a stationary observer **at a certain point in space-time**) (Barukčić, 2019). We may simply deny the existence of objective or of any other contradictions. Furthermore, even if it remains especially according to Einstein's special theory of relativity that it is not guaranteed that the notion of an absolute contradiction is justified, Einstein's special theory of relativity insist that contradictions are objective and real. That this is so highlights the fact that from the standpoint of a co-moving observer, under certain circumstances, **a path is a straight line** and nothing else. However, under the same circumstances of special theory of relativity where the relative velocity $v > 0$, from the standpoint of a stationary observer **the same path is a not a straight line, the path is curved**. The justified question is, why should and how can an identical be a contradictory too?

3.2.3 Axiom III. Lex negationis

Axiom 3. Lex negationis.

$$\neg (0) \times (+0) \equiv (+1) \tag{50}$$

where $\neg$ denotes the (natural/logical) process of negation.

III. Results

Theorem 3.1 (The geometrical form of the stress-energy tensor of the electromagnetic field $\mathbf{b}_{\mu\nu}$).

Within the frame of Einstein's theory of general (Einstein, 1916) relativity the geometrization of the electromagnetic fields has been left behind as an unsolved problem. Many different trials proposed its own way to extend the geometry of general relativity that would, so it seemed, serve as a geometrization of the electromagnetic field as well. However, the conceptual differences between the geometrized gravitational field and the classical Maxwellian theory of the electromagnetic filed were so far insurmountable.

Claim.
In general, the completely geometrical form of the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ is given by

$$b_{\mu\nu} \equiv \frac{1}{4 \times 4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \times g_{\mu\nu} \tag{51}$$

Proof By Modus Ponens.
If the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{52}$$

is true, **then** the conclusion

$$b_{\mu\nu} \equiv \frac{1}{4 \times 4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}} \right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \times g_{\mu\nu} \tag{53}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{54}$$

is true. Multiplying this premise by the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$, we obtain

$$(+1) \times b_{\mu\nu} \equiv (+1) \times b_{\mu\nu} \tag{55}$$

or

$$b_{\mu\nu} \equiv b_{\mu\nu} \tag{56}$$

Rearranging equation according to the definition 3.10 it is

$$b_{\mu\nu} \equiv \left(\frac{1}{4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F_{\nu\,\mathrm{d}} \times g^{\mathrm{cd}} \right) - \left(\frac{1}{4} \times g_{\mu\nu} \times F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \right) \right) \tag{57}$$

Rearranging equation before again it is

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi} \times \left(\left(\left(\frac{4 \times D}{4 \times D} \times \left(F_{\mu\,\mathrm{c}} \times F_{\nu\,\mathrm{d}} \times g^{\mathrm{cd}} \right) \right) - \left(\left(\frac{D}{4 \times D} \times F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \times g_{\mu\nu} \right) \right) \right) \tag{58}$$

where D denotes the number of space-time dimensions. Rearranging the equation before, we obtain

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi \times 4 \times D} \times \left(\left(4 \times D \times \left(F_{\mu\,\mathrm{c}} \times F_{\nu\,\mathrm{d}} \times g^{\mathrm{cd}} \right) \right) - \left(D \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \times g_{\mu\nu} \right) \right) \tag{59}$$

Under conditions where $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ (definition 3.14) equation before simplifies as

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi \times 4 \times D} \times \left(\left(4 \times \left(g_{\mu\nu} \times g^{\mu\nu} \right) \times \left(F_{\mu\,\mathrm{c}} \times F_{\nu\,\mathrm{d}} \times g^{\mathrm{cd}} \right) \right) - \left(D \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \times g_{\mu\nu} \right) \right) \tag{60}$$

or as

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi \times 4 \times D} \times \left(\left(\left(4 \times \left(g^{\mu\nu} \right) \times \left(F_{\mu\,\mathrm{c}} \times F_{\nu\,\mathrm{d}} \times g^{\mathrm{cd}} \right) \right) \times g_{\mu\nu} \right) - \left(D \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}} \right) \times g_{\mu\nu} \right) \right) \tag{61}$$

A further simplification of the relationship before yields the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ determined by the metric tensor of general relativity $g_{\mu\nu}$ as

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi \times 4 \times D} \times \left(\left(4 \times \left(F_{\mu\,\mathrm{c}} \times g^{\mu\nu} \times g^{\mathrm{cd}} \times F_{\nu\,\mathrm{d}}\right)\right) - \left(D \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \times g_{\mu\nu} \tag{62}$$

However, the term $\left(\left(F_{\mu\,\mathrm{c}} \times g^{\mu\nu} \times g^{\mathrm{cd}} \times F_{\nu\,\mathrm{d}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)$ of the equation before can be simplified further. For an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices (see Kay, 1988) raises each index. In other words, according to the definition 3.15 it is in general $F^{\left(\begin{smallmatrix}1 & 3\\ \mu & c\end{smallmatrix}\right)} \equiv g^{\left(\begin{smallmatrix}1 & 2\\ \mu & \nu\end{smallmatrix}\right)} \times g^{\left(\begin{smallmatrix}3 & 4\\ c & d\end{smallmatrix}\right)} \times F_{\left(\begin{smallmatrix}\nu & d\\ 2 & 4\end{smallmatrix}\right)}$ or more professionally $F^{\mu\,\mathrm{c}} \equiv g^{\mu\nu} \times g^{\mathrm{cd}} \times F_{\nu\,\mathrm{d}}$ which simplifies the term above as

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi \times 4 \times D} \times \left(\left(4 \times \left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right)\right) - \left(D \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \times g_{\mu\nu} \tag{63}$$

This relationship simplifies further as

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi} \times \left(\left(\frac{4 \times \left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right)}{4 \times D}\right) - \left(\frac{D \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)}{4 \times D}\right)\right) \times g_{\mu\nu} \tag{64}$$

Under conditions of D space-time dimensions, the geometrical form of the stress-energy momentum tensor of the electromagnetic field $\mathrm{b}_{\mu\nu}$ follows as

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi} \times \left(\left(\frac{\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right)}{D}\right) - \left(\frac{\left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)}{4}\right)\right) \times g_{\mu\nu} \tag{65}$$

According to lemma 3.2 it is $S \equiv \left(\frac{R}{D}\right)$ or $\left(\frac{1}{D}\right) \equiv \left(\frac{S}{R}\right)$. Equation before simplifies further as

$$b_{\mu\nu} \equiv \frac{1}{4 \times \pi} \times \left(\left(\frac{4 \times S \times \left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right)}{4 \times R}\right) - \left(\frac{R \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)}{4 \times R}\right)\right) \times g_{\mu\nu} \tag{66}$$

or as

$$b_{\mu\nu} \equiv \frac{1}{4 \times 4 \times \pi} \times \left(\left(4 \times S \times \left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right)\right) - \left(R \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \times \frac{g_{\mu\nu}}{R} \tag{67}$$

Following Barukčić (see Barukčić, 2016a, equation 13), it is $n\left(g_{\mu\nu}\right) \equiv \frac{g_{\mu\nu}}{R}$ In other words, equation 67 simplifies as

$$b_{\mu\nu} \equiv \frac{1}{4 \times 4 \times \pi} \times \left(\left(4 \times S \times \left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right)\right) - \left(R \times \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right)\right) \times n\left(g_{\mu\nu}\right) \tag{68}$$

Under conditions of D = 4 space-time dimensions, equation 65 simplifies further and the geometrical form of the stress-energy momentum tensor of the electromagnetic field $\mathrm{b}_{\mu\nu}$ follows as

$$b_{\mu\nu} \equiv \frac{1}{4 \times 4 \times \pi} \times \left(\left(F_{\mu\,\mathrm{c}} \times F^{\mu\,\mathrm{c}}\right) - \left(F_{\mathrm{de}} \times F^{\mathrm{de}}\right)\right) \times g_{\mu\nu} \tag{69}$$

The stress-energy momentum tensor of the electromagnetic field is geometrized completely, our conclusion is true.
Quod erat demonstrandum.

Theorem 3.2 (The geometrical form of the stress-energy tensor $T_{\mu\nu}$).

The starting point of Einstein's theory of general relativity is that gravity as such is a property of space-time geometry. Consequently, Einstein published a geometric theory of gravitation (Einstein, 1916) while Einstein's initial hope to construct a purely geometric theory of gravitation in which even the sources of gravitation themselves would be of geometric origin has still not been fulfilled. Einstein's field equations have a source term, the stress-energy tensor of matter, radiation and vacuum et cetera, which is of order two and is still devoid of any geometry and free of any geometrical significance.

Claim.

In general, the completely geometrical form of the stress-energy momentum tensor of Einstein's theory of general relativity is given by

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T \times g_{\mu\nu} \tag{70}$$

Proof By Modus Ponens.

If the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{71}$$

is true, **then** the following conclusion

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times g_{\mu\nu} \times T \tag{72}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{73}$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$(+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv (+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{74}$$

or

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{75}$$

and equally

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{D}{D} \times T_{\mu\nu} \tag{76}$$

where D denotes the number of space time dimensions. Under conditions where $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ (definition 3.14, equation 43) the equation before simplifies as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4 \times D}\right) \times (g_{\mu\nu} \times g^{\mu\nu}) \times T_{\mu\nu} \tag{77}$$

or as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4 \times D}\right) \times g_{\mu\nu} \times (g^{\mu\nu} \times T_{\mu\nu}) \tag{78}$$

In accordance with the definition 3.3 the geometrical representation of the stress-energy momentum tensor follows as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4 \times D}\right) \times T \times g_{\mu\nu} \tag{79}$$

or as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{80}$$

In accordance with definition 3.1, equation 20 it is $E \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right)$. The geometrical representation of the stress-energy momentum tensor under conditions of D dimensions follows as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv E \times g_{\mu\nu} \tag{81}$$

Under conditions of D = 4 number of space-time dimensions it is

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{82}$$

In other words, the conclusion that is true.
Quod erat demonstrandum.

Remark. Equation 80 has been derived as $\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu}$. The most simple geometrical form of the pure stress–energy momentum tensor $T_{\mu\nu}$ follows as

$$T_{\mu\nu} \equiv \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{83}$$

Lemma 3.3. *It is*

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \times g_{\mu\nu} \tag{84}$$

Simplifying equation before, the most simple geometrical form of the pure stress–energy momentum tensor $T_{\mu\nu}$ under conditions of D dimensions is determined by the equation

$$T_{\mu\nu} \equiv \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{85}$$

□

In more detail, under conditions of D = 4 dimensions the the pure stress–energy momentum tensor $T_{\mu\nu}$ is determined by the metric, enriched only by view constants and a scalar as

$$\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{86}$$

However, describing the fundamental stress–energy momentum tensor $T_{\mu\nu}$, the source term of the gravitational field in Einstein's general theory of relativity, as an inherent geometrical structure, as being determined and dependent on the metric field $g_{\mu\nu}$ is associated with several and far reaching consequences. The properties of energy, momentum, mass, stress et cetera need no longer to be seen as intrinsic properties of matter. Theoretically, the properties which material systems posses could be determined in virtue of their relation to space-time structures too. The question could arise whether the energy tensor $T_{\mu\nu}$ at the end could be in different aspects less fundamental than the metric field $g_{\mu\nu}$ itself. Is and why is matter more fundamental (Lehmkuhl, 2011; Lehmkuhl, 2014) than space-time? In contrast to such a position, is the assumption justified that **without** the space-time structure encoded in the metric **no** energy tensor? To bring it to the point, can space-time (and its geometric structure) exist without matter and if yes, what kind of existence could this be? Einstein's starting point was to derive space-time structure from the properties of material systems. In contrast to this position, theorem 3.2 allow us to see that, on the contrary, the energy tensor depend on the metric field and is completely determined by the metric field. Consequently, the matter fields themselves are derivable from the structure of space-time or the very definition of an energy tensor is determined by space-time structures too. Thus far, the question is not answered definitely, which came first, either space-time structure or energy tensor.So it is reasonable to ask, is the energy-momentum tensor of matter only dependent on the structure of space-time or even determined by the structure of space-time or both or none? In other words, granddaddies **either chicken or the egg** dilemma is asking for an innovative and a comprehensive solution and may end up in an Anti-Machian theory. However, this leads us at this point too far afield.

Theorem 3.3 (Einstein's field equation's completely geometrized).

Now, we can derive a completely geometrical form of Einstein field equations under conditions of D dimensions.

Claim.

In general, the completely geometrical form of Einstein field equation's (Einstein, 1916; Einstein & de Sitter, 1932; Einstein, 1917) under conditions of D dimensions is given by

$$(S \times g_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv E \times g_{\mu\nu} \tag{87}$$

Proof By Modus Ponens.

If the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{88}$$

is true, **then** the following conclusion

$$(S \times g_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv E \times g_{\mu\nu} \tag{89}$$

is also true, again the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{90}$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$(+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv (+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{91}$$

or

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times 4 \times T_{\mu\nu} \tag{92}$$

Rearranging equation according to the definition 3.2 it is

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{93}$$

Taking the trace with respect to the metric of both sides of the Einstein field equations one gets

$$R_{\mu\nu} \times g^{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + (\Lambda \times g_{\mu\nu} \times g^{\mu\nu}) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \tag{94}$$

Equation 94 simplifies as

$$R - \left(\left(\frac{R}{2}\right) \times D\right) + (\Lambda \times D) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T \tag{95}$$

Dividing equation 95 by the number of dimensions D, it is

$$\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{96}$$

where D is the space-time dimension. In point of fact, due to lemma 3.2, equation 39, it is $S \equiv \left(\frac{R}{D}\right)$. Substituting this relationship into the equation 96, we obtain

$$S - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{97}$$

According to lemma 3.1, equation 20, it is $E \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right)$ and equation 97 changes to

$$S - \left(\frac{R}{2}\right) + (\Lambda) \equiv E \tag{98}$$

The general geometrical form of Einstein field equation under conditions of D dimensions is obtained by multiplying equation 98 by the metric tensor $g_{\mu\nu}$ as

$$(S \times g_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + (\Lambda \times g_{\mu\nu}) \equiv E \times g_{\mu\nu} \tag{99}$$

The conclusion is true.
Quod erat demonstrandum.

Theorem 3.4 (**Einstein's cosmological constant** Λ).

An even more severe violation of our trust into physics is created by the cosmological constant Λ, which specifies as the overall vacuum energy density. Depending on the specific assumptions made, the physical value (Weinberg, 1987) of the cosmological constant Λ is found to be very contradictory. Now, we can calculate the value of the cosmological constant Λ very precisely.

Claim.
In general, the value of the cosmological constant Λ is given by

$$\Lambda \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) + \left(\frac{R}{2}\right) - S \tag{100}$$

Proof By Modus Ponens.
If the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{101}$$

is true, **then** the following conclusion

$$\Lambda \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) + \left(\frac{R}{2}\right) - S \tag{102}$$

is also true, again the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{103}$$

is true. Multiplying this premise by the left part of equation 97 which is $S - \left(\frac{R}{2}\right) + (\Lambda)$, we obtain

$$S - \left(\frac{R}{2}\right) + (\Lambda) \equiv S - \left(\frac{R}{2}\right) + (\Lambda) \tag{104}$$

In brief, equation 97 demands that $S - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right)$. Therefore, equation 104 changes too

$$S - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{105}$$

Rearranging equation 105 yields **the exact value of Einstein's cosmological constant** Λ **under D dimensions** as

$$\Lambda \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) + \left(\frac{R}{2}\right) - S \tag{106}$$

with the consequence that our conclusion is true.

Quod erat demonstrandum.

Remark 3.2. *The most important outcome of theorem 3.4 is the discovery that the exact value of* ***Einstein's cosmological constant*** Λ ***depends on D, the number of space-time dimensions****. It may proof as true especially by measurements that theorem 3.4 induces some reasonable doubts with respect to the constancy of Einstein's cosmological constant* Λ.

Theorem 3.5 (**Anti cosmological constant $\underline{\Lambda}$**).

The value of the anti-cosmological constant can be calculated very precisely.

Claim.

In general, the value of the anti-cosmological constant $\underline{\Lambda}$ (Einstein, 1916; Einstein & de Sitter, 1932; Einstein, 1917) is given by

$$\underline{\Lambda} \equiv S + \left(\frac{R}{2}\right) - \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{107}$$

Proof By Modus Ponens.

If the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{108}$$

is true, **then** the following conclusion

$$\underline{\Lambda} \equiv S + \left(\frac{R}{2}\right) - \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{109}$$

is also true, again the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{110}$$

is true. Multiplying this premise by Ricci scalar (see definition 3.12), we obtain

$$(+1) \times (R) \equiv (+1) \times (R) \tag{111}$$

or

$$R \equiv R \tag{112}$$

Adding Λ and subtracting Λ, the cosmological constant, it is

$$R - \Lambda + \Lambda \equiv R - \Lambda + \Lambda \tag{113}$$

or

$$R - \Lambda + \Lambda \equiv R \tag{114}$$

According to our definition 3.12 it is

$$\underline{\Lambda} + \Lambda \equiv R \tag{115}$$

and therefore

$$\underline{\Lambda} \equiv R - \Lambda \tag{116}$$

The exact value of the cosmological constant Λ under conditions of D space-time dimensions was calculated by theorem 3.4, equation 106 as $\Lambda \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) + \left(\frac{R}{2}\right) - S$. **The exact value of the anti cosmological constant** $\underline{\Lambda}$ can be calculated as

$$\underline{\Lambda} \equiv R - \left(\left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) + \left(\frac{R}{2}\right) - S\right) \tag{117}$$

or as

$$\underline{\Lambda} \equiv R - \left(\frac{R}{2}\right) - \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) + S \equiv S + \left(\frac{R}{2}\right) - \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{118}$$

with the consequence that the conclusion is true.

Quod erat demonstrandum.

IV. Discussion

Einstein was one of the first to use explicitly the term "**unified field theory**"in the title (Einstein, 1925) of a publication in 1925. In the following, Einstein himself published more than thirty technical papers on this topic. However, Einstein's unified field theory program, besides of his justified insistence on the possibility and desirability of a unified field theory, required a substantial amount of new mathematical preliminaries and methods (Barukčić, 2016a, 2016c) and was on the level of the mathematical possibilities at his time technically in vain.
At the heart of this enterprise was the trial to geometrize all fundamental interactions and to provide a completely geometrized (Einstein, 1950) theory of relativity was endangered by the cosmological constant Λ, the energy density of space, or vacuum energy, and the uncertainties associated with the same. To day, there is some experimental evidence (Perlmutter et al. (Perlmutter et al., 1999) *Supernova Cosmology Project* and Riess et al. (Riess et al., 1998) *High-Z Supernova Search Team*) that the expansion of the universe is accelerating, implying the possibility of a positive nonzero value for the cosmological constant Λ. Considered Einstein's insight (see Einstein, 1916, p. 796) that $g_{\mu\nu} \times g^{\mu\nu} \equiv D = +4$ (definition 3.14) it was possible to geometrize the stress-energy tensor of Einstein's general theory of relativity even under D space-time dimensions. Encouraged by this result, it was possible to calculate **the exact value of the cosmological constant Λ under conditions where** $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ (definition 3.14). However, an answer to the question whether the condition $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ is generally given may predominantly be found elsewhere. Under conditions where $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ where D is the number of space-time dimensions, we are able to calculate the exact value of the cosmological constant Λ very precisely as

$$\Lambda \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) + \left(\frac{R}{2}\right) - S$$

and much more than this. For the reasons set out above, the inevitable conclusion is that even the value of the anti cosmological constant $\underline{\Lambda}$ follows as

$$\underline{\Lambda} \equiv S + \left(\frac{R}{2}\right) - \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right).$$

V. Conclusion

In combination with other already published (Barukčić, 2016a, 2016c) papers, Einstein's general theory of relativity is completely geometrized. The theoretical value of the cosmological constant Λ was calculated very precisely.

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