

ZERO DIVIDED BY ZERO DOES NOT EQUAL ZERO

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ABSTRACT. Aim: Indeterminate forms are still not accepted as solved by the majority of the scientific community.

Methods: Therefore, indeterminate forms like $\frac{+0^{+1}}{+0^{+1}} \equiv +0^{+1}$ and $\frac{+0^{+1}}{+0^{+1}} \equiv +\infty^{+1}$ have been re-analysed again.

Results: We must accept that $\frac{+0^{+1}}{+0^{+1}} \neq +0^{+1}$ and $\frac{+0^{+1}}{+0^{+1}} \neq +\infty^{+1}$

Conclusion: Zero divided by zero equals one.

1. INTRODUCTION

The topic of **the unity of science** includes many different questions. With respect to the wholeness of science one of the challenges is to answer the question whether the various natural sciences (e.g., mathematics, physics, astronomy, chemistry, biology) can be unified into a single overarching theory. Since these sciences explore objective reality as it is, as **existing independently and outside of human mind and consciousness**, the question is justified too, is the unity of nature or are the principles and laws of the self-organisation of matter the foundation of human science too. There are many principles of science. One of this basic principles of science is the principle of contradiction. Following Karl Raimund Popper (1902 - 1994),

“A theory which involves a contradiction
is ... entirely useless
as a theory”
[57]

The principle of contradiction is one of the common foundations of human science. Therefore, a reliable and clear scientific methodology should be able to help us to decide what is true and what is false[7] and to assure a kind of **a demarcation line between science and non-science** [57]. Who really knows the further

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course of scientific development. Many times, science itself is like riding a wild horse driven by its instincts without a stop or a chance to take a break. In this context,

**“Any intelligent fool can
make things bigger, more complex, and more violent.
It takes a touch of genius —
and a lot of courage
to move in the opposite direction.”**
[67]

However, to maintain the balance perfectly and not to be thrown off it is necessary to keep ‘things’ as simple as possible.

**“... the supreme goal of all theory is to
make the irreducible basic elements
as simple and as few as possible ...”**
[28]

2. MATERIAL AND METHODS

From a practical point of view, various proposals have been put forward which criteria of demarcation between science and non-science should be applied, including **modus tollens** as advocated especially by **Karl Popper**. Following Popper,

**“... it is possible by means of purely deductive inferences
(with the help of the modus tollens of classical logic)
to argue
from the truth of singular statements
to the falsity of universal statements.”**
[56]

However, **modus inversus** is an additional approach to solve the problem of demarcation between science and non-science. In contrast to **modus ponens**, **modus inversus is designed primarily to preserve at all costs the contradiction**, the falsity, the falseness, the falsehood as such. In contrast to the principle ex contradictione sequitur quodlibet [19, 58, 59], **from a contradictory premise or a contradictory statement like $(+1=+0)$, does not anything follow but the contradiction itself**. In other words, in the absence of errors, **the contradiction is preserved**. In particular, even if one of the main tasks of modus inversus [6] is preserve the contradiction under any circumstances, the main task of modus inversus is to recognise the truth too. The abstract structure of modus inversus will be discussed here and may be found in literature too.

2.1. Definitions. Reaching a generally valid consensus on the definition of the numbers $+0$ and $+1$ appears to be difficult. These numbers are fundamental importance in classical logic, probability theory and so forth. The definition of the basic numbers $+1$ and $+0$ in terms of Euler's identity and physical 'constants' offer us the possibility to test classical logic or mathematical theorems et cetera by reproduce-able physical experiments too. In particular, it is very remarkable that Leibniz [44] himself published in 1703 the first self-consistent binary number system representing all numeric values while using typically $+0$ (zero) and $+1$ (one).

2.1.1. The number $+0$.

Definition 2.1 (The number $+0$). Let c denote the speed of light in vacuum [25, 70, 75, 76], let ϵ_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary number [13]. The number $+0$ is defined as the expression

$$(2.1) \quad +0 \equiv +1 - 1 \equiv +1 + i^2 \equiv +1 + e^{i\pi} \equiv + (c^2 \times \epsilon_0 \times \mu_0) + e^{i\pi}$$

while '=' or \equiv denotes the equals sign [61] or equality sign [63] used to indicate equality and '-' [50, 78] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and '+' denotes the plus [61] signs used to represent the operations of addition and the notions of positive as well.

Remark 2.1. Roger Cotes (1682 – 1716) [21] or Leonhard Euler's (1707 – 1783) identity [29] is regarded as one of the most beautiful equations [79]. In this context, it is provisionally presumed, that Euler's identity [29] is logically sound and correct.

2.1.2. The number $+1$.

Definition 2.2 (The number $+1$). Again, let c denote the speed of light in vacuum [25, 70, 75, 76], let ϵ_0 denote the electric constant and let μ_0 the magnetic constant. Let i denote the imaginary number [13]. The number $+1$ is defined as the expression

$$(2.2) \quad +1 \equiv +1 + 0 \equiv +1 - 0 \equiv -i^2 \equiv -e^{i\pi} \equiv + (c^2 \times \epsilon_0 \times \mu_0)$$

while again '=' or \equiv may denote the equals sign [61] or equality sign [63] used to indicate equality and '-' [50, 78] denotes minus signs used to represent the operations of subtraction and the notions of negative as well and '+' denotes the plus [61] signs used to represent the operations of addition and the notions of positive as well.

2.1.3. The infinity.

Definition 2.3 (The infinity). Let $+\infty$ denote positive infinity. Let $-\infty$ denote negative infinity.

Infinity may possess many properties while only a few of these properties are identified by science. However, Einstein calls us for caution in this respect.

“Two things are infinite,
the universe
and
human stupidity,
and I am not yet completely sure about the universe.”
[54]

2.1.4. *Basic rules for exponentiation.*

Definition 2.4 (Basic rules for exponentiation). Let n denote a positive integer and let x denote any real number, then x^n corresponds to repeated multiplication as

$$(2.3) \quad x^n \equiv \underbrace{x \times x \times x \times x \dots}_{n\text{-times}} \equiv \underbrace{x^1 \times x^1 \times x^1 \times x^1 \dots}_{n\text{-times}} \equiv x^{1+1+1+1+\dots}$$

From this definition, some basic rules of exponentiation can be deduced. It is

$$(2.4) \quad x^a \times x^b \equiv x^{a+b}$$

or

$$(2.5) \quad (x \times y)^a \equiv x^a \times y^a$$

et cetera.

2.1.5. *Zero divided by zero equals zero.*

Definition 2.5 (Zero divided by zero equals zero). Independently of the evidence [3–5, 8, 51–53, 72] provided to the contrary, several [1, 15, 18, 49, 65]

authors published positions with respect to zero [15, 39, 65] while view of them continue [11, 12, 16, 42, 45, 47, 55] repeatedly to reinforce in a number of different ways statements which demand us to accept that

$$(2.6) \quad \frac{+1}{+0} \equiv \frac{+0}{+0} \equiv +0$$

2.2. Proof methods. Considered from the historical point of view, human reasoning and knowledge appears to be to some extent relative too. Although it seems almost impossible, to proof or to establish the correctness of a statement, a theorem, a theory once and for all, this does not justify any technical or other errors in (human) reasoning which are many times identified the hard way but easy to overlook while in contrast to that **charges and proofs of fallacious reasoning always need time, money, and personal dignity to be accepted by the scientific community.**

“Niemals aber kann die Wahrheit einer Theorie erwiesen werden.
Denn niemals weiß man,
daß auch in Zukunft eine Erfahrung bekannt werden wird,
die Ihren Folgerungen widerspricht...”

[26]

Albert Einstein's position translated into English: 'But the truth of a theory can never be proven. For one never knows if future experience will contradict its conclusion; and furthermore there are always other conceptual systems imaginable which might coordinate the very same facts. 'Often, our fear of the unknown appears to overshadow our mind to an objectively unjustified extent. However, logically sound scientific verification and proof techniques are likely to allow us to continue our successful and rapid identification of contradictory scientific findings and are appropriate enough to shed some light even on this unknown. Step by step, by following the time honoured principle of going **from the known (and secured) to the unknown (and unsecured)** we will bring more light into the epistemological darkness which may surround us sometimes. Following Einstein, a theory can very well be found to be incorrect if there is a logical error in its deduction.

**“Eine Theorie kann also wohl als unrichtig erkannt werden,
wenn in ihren Deduktionen
ein logischer Fehler ist ...”**

[26]

In other words, grain by grain and the hen fills her belly. Scientific proof methods are a **demarkation line between science and non-science** [57]. In this context, the development of new suitable scientific experimental and non-experimental test methods is of key scientific value. It may be allowed to point out view of these numerous scientific proof [6] methods.

2.2.1. *Proof by counter example.*

Definition 2.6 (Proof by counter example). Scientific progress can be achieved not only through doing things right, but also by correcting (scientific) mistakes. Both contributions of authors are equivalent to each other and the two sides of the same coin. A **proof by counter example** is a valid scientific proof technique with the potential to correct horrific and dreadful scientific mistakes especially in philosophy, mathematics and science as such.

**“No amount of experimentation
can ever prove me right;
a single experiment
can prove me wrong.”**

[62]

In particular, the close investigation of counter examples can give us an insight into the many deep and delicate issues surrounding a statement or theorem. A generally valid theorem can be refuted by a single counter example [10, 20, 38, 46, 62, 64, 69, 74] by showing an instance where a given statement, theorem et cetera cannot possibly be correct.

It is worth to emphasise in this context that **one single counter example** refutes a theorem, a theory, a conjecture **as effectively as n** counter examples.

2.2.2. *Proof by (thought) experiments.* Unfortunately, too often, competing scientific positions or even theories of the nature or of our world are excluding each other. A (theoretical) scientific verification becomes pressing while (thought) experiments are of special importance in this context. In short, Albert Einstein wrote in a letter to the student J. S. Switzer on April 23th, 1953, **Albert Einstein** the following:

“Development of Western **science** is based on two great achievements: the invention of the formal **logical** system (in Euclidean geometry) by the Greek philosophers, **and** the discovery of the possibility to find out **causal relationships** by systematic **experiment** (during the Renaissance). ”
[37]

In other words, (thought) experiments are one of the methods to proof theorems and theories.

2.2.3. *Modus tollens.* From a practical point of view, various proposals [9] have been put forward which criteria of demarcation between science and non-science should be applied, including **modus tollens** as advocated especially by **Karl Popper**. Following Popper,

“... it is possible by means of purely deductive inferences
(with the help of the modus tollens of classical logic)
to argue
from the truth of singular statements
to the falsity of universal statements.”
[56]

2.2.4. *Proof by modus inversus.* It is noticeable that our today’s methods of investigation especially in natural sciences and even the knowledge achieved relies to a very great extent on mathematics and mathematical rules too. Thus far, mathematics as such appears to enjoy a very special esteem within scientific community and is regarded more or less as above all other sciences [9]. This view is sometimes further strengthened by the common believe that the laws or mathematics are absolutely certain and indisputable. However, it is noteworthy that objects studied in mathematics are not all the time located in space and time and the methods of

investigation of mathematics differ sometimes markedly from the methods of investigation in the natural sciences [9]. Therefore, first and after all and in a slightly different way, **today's mathematics itself is more or less a product of human thought and mere human imagination** and belongs as such to a world of human thought and mere human imagination. In point of fact, **human thought and mere human imagination which produces the laws of mathematics is able to produce erroneous or incorrect results too** with the principal consequence that even mathematics or **mathematical theorems, rules or other results valid since thousands of years are in constant danger of being overthrown by newly discovered facts** [9]. **Modus inversus** [6, 9, 71] is a suitable proof method to check mathematical position and theorems for logical consistency.

However, **modus inversus** is an additional approach to solve the problem of (see also: <https://doi.org/10.5281/zenodo.4165074>) demarcation between science and non-science. In contrast to **modus ponens**, **modus inversus is designed primarily to preserve at all costs the contradiction**, the falsity, the falseness, the falsehood as such. In contrast to the principle *ex contradictione sequitur quodlibet* [19, 58, 59], **from a contradictory premise or a contradictory statement like (+1=+0), does not anything follow but the contradiction itself**. In other words, in the absence of (technical and other) errors, **the contradiction is preserved**. In particular, even if one of the main tasks of **modus inversus** [6] is to preserve the contradiction under any circumstances, the main task of **modus inversus** is to recognise the truth too. The abstract structure of **modus inversus** is as follows.

Proof by modus inversus. Thus far, let ${}_R P_t$ denote a premise at a certain point in (space-) time t . Let ${}_R C_t$ denote the conclusion at the same certain point in (space-) time t .

Premises.

- (1) If (${}_R P_t$ is false) then (${}_R C_t$ is false).
- (2) ${}_R P_t$ is false.

Conclusion.

- (3) ${}_R C_t$ is false. □

The following 2x2 table may illustrate **modus inversus** again. Let ${}_R P_t$ denote a premise from the standpoint of a stationary observer, a Bernoulli distributed random variable at a certain period of time or Bernoulli trial t [73].

TABLE 1. Modus inversus

		Conclusion ${}_R C_t$	
		FALSE	TRUE
Premisse ${}_R P_t$	FALSE	+1	+0
	TRUE	+1	+1
			+1

In terms of probability theory **modus inversus** can be expressed as follows.

TABLE 2. Modus inversus II

		Conclusion ${}_R C_t$		
		FALSE	TRUE	
Premisse ${}_R P_t$	FALSE	$p(a_t)$	+0	$p({}_R P_t)$
	TRUE	$p(c_t)$	$p(d_t)$	$p({}_R P_t)$
		$p({}_R C_t)$	$p({}_R \underline{C}_t)$	+1

The premise takes only the values ${}_R P_t$ either +0 or +1. Let ${}_R C_t$ denote a conclusion from the standpoint of a stationary observer R, a Bernoulli distributed random variable at the same period of time or Bernoulli trial t. The conclusion ${}_R C_t$ itself can take only the values ${}_R C_t$ either +0 or +1. Under conditions of classical logic, +0 may denote false while +1 may denote true. The modus inversus is defined as if (premise_t is false) then (conclusion_t is false). Formally, modus inversus can be expressed as

$$(2.7) \quad ({}_R P_t) \cup ({}_R \neg C_t) \equiv +1$$

while the sign \cup denotes inclusive or. It is noticeable and by far not regrettable that according to modus inversus **it is not possible to achieve a true conclusion while starting with a false premise**. The follow-up question should be: what allows the assumption that modus inversus is generally valid or valid at all?

Example: Burning candle experiment

A simple to perform real-world experiment may illustrate the general validity of modus inversus. Let A_t denote gaseous oxygen, a Binomial random variable, which can take only two values, either gaseous oxygen present = +1 or gaseous oxygen not present = +0. Gaseous oxygen is present means that the amount of gaseous oxygen is enough to assure that a candle can burn. Let B_t denote a candle, a Binomial random variable, which can take only two values, either a candle is burning = +1 or a candle is not burning = +0.

In this experiment, an investigator lights the candle wick of some candles (old, you, big, small, red, green, curved, straight et cetera) under different conditions. As next, candle flame reacts with gaseous oxygen such that light and heat which characterises a candle are produced. The data as obtained by this real world experiment are illustrated by the following 2x2 table.

TABLE 3. Example. Modus inversus III

		Candle is burning	
		FALSE	TRUE
Gaseous oxygen	FALSE	+1	+0
	TRUE	+1	+1
		+1	

The relationship between gaseous oxygen and the behaviour of a candle produced out of simple wax is studied to demonstrate the relationship of modus inversus to

objective reality. In other words, *modus inversus* is backed by natural processes independent of human mind and consciousness.

For this reason, and especially if different persons with different ideology and believe are aiming to arrive at the same logical conclusions with regard to such a difficult topic as indeterminate forms are, they will have to agree at least upon some view fundamental laws (axioms) as well as the methods by which other laws can be deduced therefrom. At this point, clarifying some fundamental axioms or starting points of investigations can therefore be essential part of every scientific method and any scientific progress.

2.2.5. *Direct proof.* The truth or falsehood of a given theorem can be demonstrated too by a straightforward combination of established facts.

2.2.6. *Proof by contradiction.* Proof by contradiction [24, 80] is a widely used proof method and goes back at least as far as to ancient times. The truth or the validity of a theorem can be established by **assuming that a statement or a theorem we want to prove is false**. In the following of the proof by showing that such an assumption leads to a contradiction it is justified to conclude that we were wrong to assume the theorem was false. In other words, the theorem must be true.

2.2.7. *Proof by other methods.* There are of course many other scientific proof methods which can be found in literature.

2.3. **Axioms.** One of the goals of science is that different persons should arrive at the same logical conclusions independently of any ideology and subjective motives after they have already agreed upon the fundamental axioms (laws) of a theory, as well as the (proof, experimental and other) methods or rules by which other laws are to be deduced therefrom. On this head, Einstein himself notes in particular in this regard - almost unnoticed - brilliantly

“Denn es kann nicht wundernehmen, wenn man zu **übereinstimmenden logischen Folgerungen** kommt, nachdem man sich **über die fundamentalen Sätze (Axiome) sowie über die Methoden geeinigt** hat, vermittels welcher **aus diesen fundamentalen Sätzen andere Sätze abgeleitet** werden sollen. ”

[27]

Mathematics and mathematical rules plays a very big part in today's science. Einstein's comments rightly: 'One reason why mathematics enjoys special esteem, above all other sciences, is that its laws are absolutely certain and indisputable, while those of all other sciences are to some extent debatable and in constant danger of being overthrown by newly discovered facts.' Einstein's original position in German language:

“Die **Mathematik**
 genießt vor allen anderen Wissenschaften aus einem Grunde ein besonderes
 Ansehen;
 ihre Sätze sind
absolut sicher und unbestreitbar,
 während die aller andern Wissenschaften
 bis zu einem gewissen Grad umstritten und stets in Gefahr sind,
 durch neuentdeckte Tatsachen umgestoßen zu werden. ”
 [27]

In the light of the importance of mathematics and mathematical rules for achieving new research insights and to disseminate scientific knowledge to a wider public in a generally accepted and understandable form, it is necessary to take into account that many times the insights are gained in terms of the validity of the assumption that mathematical rules themselves are logically consistent.

“Aber jenes große Ansehen der Mathematik ruht andererseits darauf, daß die
 Mathematik es auch ist, die den exakten Naturwissenschaften ein gewisses Maß
 von Sicherheit gibt, das sie ohne Mathematik nicht erreichen könnten. ”
 [27]

Einstein’s position translated into English: ‘But that big one reputation of mathematics rests on the other hand in that it is also mathematics which affords the exact natural sciences a certain degree of certainty, to which without mathematics they could not achieve.’ To date and with all due respect and enthusiasm for mathematics, one should keep in mind that mathematics as such and the laws and the rules of mathematics are after all a product of pure human thought mere human imagination and as such independent of human experience and real world experiments. Whether human reason as such does determine the properties of real things existing independently and outside of human mind and consciousness may be discussed somewhere else but it is for sure not for the mathematician to decide. To many times, mathematical laws and rules are not always verified or checked definitely or the right way. We must carefully bear in mind that especially the laws and rules of mathematics are not absolutely certain and not indisputable, and are to a very great extent debatable and in constant danger of being overthrown by newly discovered facts. Nonetheless, given the reservations and reticence expressed concerning the need to ensure that mathematics itself should be logically consistent demand us to take care that any mathematical drill and blind obedience to mathematical rules, however convincing the same may be, is not justified as long as the same rules are not proofed as logically consistent. In this respect, we find ourselves in almost complete harmony with Albert Einstein who wrote:

“I don’t believe in mathematics.”

[17]

2.3.1. *Axiom I. Lex identitatis.* Backed by thousands of years of often bitter human experience, the scientific development has taught us all that human knowledge is relative too. Even if experiments and other suitable proofs are of help to encourage us more and more in our belief of the correctness of a theory, it is difficult to proof the correctness of a theorem or of a theory et cetera once and for all.

**“Niemals aber kann die Wahrheit einer Theorie erwiesen werden.
Denn niemals weiß man,
daß auch in Zukunft eine Erfahrung bekannt werden wird,
die Ihren Folgerungen widerspricht...”**

[26]

Albert Einstein’s position translated into English: ‘But the truth of a theory can never be proven. For one never knows if future experience will contradict its conclusion; and furthermore there are always other conceptual systems imaginable which might coordinate the very same facts. ’However, another remark of Einstein is worth being considered.

**“No amount of experimentation
can ever prove me right;
a single experiment
can prove me wrong.”**

[62]

In the light of the foregoing it is clear that appropriate axioms and conclusions derived from the same are a main logical foundation of any ‘theory’.

**“Grundgesetz (Axiome)
und
Folgerungen
zusammen bilden das was man
eine ‘Theorie’
nennt. ”**

[26]

Albert Einstein's (1879-1955) message translated into English as: *Basic law (axioms) and conclusions together form what is called a 'theory'* has still to get round. However, an axiom as a free creation of the human mind which precedes all science should be like all other axioms, as simple as possible and as self-evident as possible. In this context, we define axiom I as

$$(2.8) \quad + 1 = +1$$

Historically, Aristotle himself already cited **the law of excluded middle** and **the law of contradiction** as examples of axioms. However, **lex identitatis** is an axiom too, which possess the potential to serve as the most basic and equally as the most simple axiom of science. Something which is really just itself is equally different from everything else. In point of fact, is such an equivalence which everything has to itself inherent or must the same be constructed by human mind and consciousness. Can and how can something be **identical with itself**[31, 33, 40, 48] and in the same respect different from itself. An increasingly popular view on identity is the one advocated by Gottfried Wilhelm Leibniz (1646-1716):

“Chaque chose est ce qu'elle est.
Et dans autant d'exemples qu'on voudra
A est A,
B est B. ”
[43]

or $A = A$, $B = B$ or $+1 = +1$. Exactly in complete compliance with Leibniz, Johann Gottlieb Fichte (1762 - 1814) elaborates on this subject as follows:

“Each thing is what it is ;
it has those realities which are posited when it is posited,
($A = A$.) ”
[30]

2.3.2. *Axiom II. Lex contradictionis.* In this context, axiom II or **lex contradictionis**, the negative of lex identitatis, or

$$(2.9) \quad + 0 = +1$$

and equally the most simple form of a contradiction formulated. Thus far, axiom II is of no minor importance too. Scientist inevitably have false beliefs and make mistakes. In order to prevent scientific results from falling into logical inconsistency or logical absurdity, it is necessary to possess among other the methodological possibility to start a reasoning with a contradiction too. However and in contrast to the way of reasoning with inconsistent premises as proposed by para-consistent logic [19, 22, 23, 58–60], in

the absence of technical and other errors of reasoning, the contradiction itself need to be preserved. In other words, **from a contradiction does not anything follows but the contradiction itself** while the theoretical question is indeed justified “What is so Bad about Contradictions?” [58]. Historically, **the principle of (deductive) explosion**, coined by 12th-century French philosopher William of Soissons, demand us to accept that anything, including its own negation, can be proven or can be inferred from a contradiction. Respecting the principle of explosion, the existence of a contradiction (or the existence of logical inconsistency) in a scientific theorem, rule et cetera is disastrous. However, the historical development of science shows that scientist inevitably revise the theories, false positions and claims are identified once and again, and we all make different kind of mistakes. In order to avert a disproportionately great damage on science and to prevent reducing science into pure subjective belief, a negation of the principle of explosion is required. Nonetheless, a justified negation of **the ex contradictione quodlibet principle** [19] does not imply the correctness of paraconsistent logic [19, 22, 23, 58–60] as such as advocated especially by the Peruvian philosopher Francisco Miró Quesada [60] and other [19, 22, 23, 58, 59]. In general, scientific theories appear to progress from lower and simpler to higher and more complex levels. However, high level theories cannot be taken for granted because high level theories are grounded on a lot of assumptions, definitions and other procedures and may rest upon too much erroneous stuff even if still not identified. Therefore, it should be considered to check at lower at simpler levels like with like.

In philosophy, **the principle of explosion** or the principle of Pseudo-Scotus (Latin: **ex contradictione (sequitur) quodlibet**) demand us to accept that from falsehood or from a contradiction, any statement can be proven or anything may follow. However, scientist inevitably have false beliefs and make mistakes. In order to prevent us from falling into logical inconsistency or logical absurdity, it is necessary to posses the possibility to start a reasoning with a contradiction too. However and in contrast to the way of reasoning with inconsistent premises as proposed by para-consistent logic [19, 22, 23, 58–60], in the absence of technical and other errors of reasoning, the contradiction itself need to be preserved. In other words, **from a contradiction does not anything follows but the contradiction itself**.

2.3.3. *Axiom III. Lex negationis.*

$$(2.10) \quad \neg(0) \times 0 = 1$$

where \neg denotes (logical [14] or natural) negation [2, 31, 32, 34–36, 40, 41, 48, 66, 68, 77]. In this context, there is some evidence that $\neg(1) \times 1 = 0$. In other words, it is $(\neg(1) \times 1) \times (\neg(0) \times 0) = 1$

3. RESULTS

3.1. Refutation of the principle of explosion.

Theorem 3.1 (Refutation of the principle of explosion). *In general, the principle of explosion is logically inconsistent and refuted.*

From a contradiction does not anything follow but the contradiction itself.

Proof by modus inversus. **If** the premise

$$(3.1) \quad \underbrace{+0 = +1}_{(Premise)}$$

is false, **then** the conclusion

$$(3.2) \quad +1 \equiv +2$$

is false too. The premise is false. Adding +1 on both sides of Eq. 3.2 yields Eq. 3.3 as

$$(3.3) \quad +1 \equiv +2$$

which is false too. In other words, from a contradiction does not anything follow but the contradiction itself and our conclusion is true. This proof can be repeated as many times as desired (i.e. ∞ -times) with the same number or with different numbers, the result will not change, a contradiction is the final result. The principle of explosion is refuted. \square

3.2. Negative times negative equals negative.

Theorem 3.2 (Negative times negative equals negative.). ***A negative times negative equals a negative. In general, it is***

Proof by modus inversus. **If** the premise

$$(3.4) \quad \underbrace{+0 = +4}_{(Premise)}$$

is false, **then** the conclusion

$$(3.5) \quad (-2)^{+2} \equiv (+2)^{+2}$$

is false too. The premise is false. Subtracting -2 on both sides of Eq. 3.4 yields Eq. 3.6 as

$$(3.6) \quad (-2) \equiv (+2)$$

Performing an arithmetic operation (exponentiation (power)) on Eq. 3.6 yields

$$(3.7) \quad (-2)^{+2} \equiv (+2)^{+2}$$

which need to be false too. In contrast to this result, today's mathematical rules demand us to accept that a negative squared changes into a positive. The centuries-old mathematical rule that **a negative times a negative** changes into a positive does not appear to be tenable any longer. \square

3.3. $\infty^{+1} \neq \infty^{+2}$.

Theorem 3.3 ($+\infty$ power +1 does not equal $+\infty$ power +2). *In general, we must accept that*

$$(3.8) \quad +\infty^{+1} \neq +\infty^{+2}$$

Proof by modus inversus. **If** the premise

$$(3.9) \quad \underbrace{+1 = +2}_{(Premise)}$$

is false, **then** the conclusion

$$(3.10) \quad +\infty^{+1} \equiv +\infty^{+2}$$

is false too. The premise is false. Performing an arithmetic operation on the premise (exponentiation (power)) yields

$$(3.11) \quad +\infty^{+1} \equiv +\infty^{+2}$$

which is false too. In other words, we must accept in general that

$$(3.12) \quad +\infty^{+1} \neq (+\infty^{+1} \times +\infty^{+1} \equiv +\infty^{+2})$$

and our conclusion is true. □

The theorem is valid for negative infinity too.

3.4. $0^{+1} \neq 0^{+2}$.

Theorem 3.4 (Zero power +1 does not equal zero power +2). *In general, we must accept that*

$$(3.13) \quad +0^{+1} \neq +0^{+2}$$

Proof by modus inversus. **If** the premise

$$(3.14) \quad \underbrace{+1 = +2}_{(Premise)}$$

is false, **then** the conclusion

$$(3.15) \quad +0^{+1} \equiv +0^{+2}$$

is false too. The premise is false. Performing an arithmetic operation on the premise (exponentiation (power)) yields

$$(3.16) \quad +0^{+1} \equiv +0^{+2}$$

which is false too. In other words, we must accept in general that

$$(3.17) \quad +0^{+1} \neq (+0^{+1} \times +0^{+1} \equiv +0^{+2})$$

and our conclusion is true. □

3.5. $0^{+1} / 0^{+1} \neq 0^{+1}$.

Theorem 3.5 (Zero divided by zero does not equal zero). *In general, we must accept that*

$$(3.18) \quad \frac{+0^{+1}}{+0^{+1}} \equiv +0^{+1-1} \equiv +0^{+0} \equiv +1^{+1} \equiv +1$$

Proof by contradiction. **The** premise

$$(3.19) \quad \underbrace{+1 = +1}_{(Premise)}$$

is true. In this case, **the** conclusion drawn need to be true too. Rearranging equation 3.19, we obtain

$$(3.20) \quad +1 - 1 \equiv +0$$

or

$$(3.21) \quad +0^{+1} \equiv +0^{+1}$$

Dividing by $+0^{+1}$, we obtain

$$(3.22) \quad \frac{+0^{+1}}{+0^{+1}} \equiv \frac{+0^{+1}}{+0^{+1}}$$

Several authors are claiming that equation 3.22 is equivalent with the following equation.

$$(3.23) \quad +0^{+1} \equiv \frac{+0^{+1}}{+0^{+1}}$$

Let us assume for preliminary reasons that this claim is true. In the following it is not allowed to deduce a contradiction out of such a claim. Rearranging equation 3.23 we obtain

$$(3.24) \quad \begin{aligned} +0^{+1} &\equiv \frac{+0^{+1}}{+0^{+1}} \\ &\equiv \frac{+1^{+1} \times +0^{+1}}{+1^{+1} \times +0^{+1}} \\ &\equiv \frac{+0^{+1} \times +1^{+1}}{+1^{+1} \times +0^{+1}} \\ &\equiv +0^{+1} \times \frac{+1^{+1}}{+0^{+1}} \end{aligned}$$

In other words, it is $\frac{+0^{+1}}{+0^{+1}} \equiv \left(+0^{+1} \times \underbrace{\frac{+1^{+1}}{+0^{+1}}}_{Something} \right)$ or zero times something and not

only zero. However, the authors claim that this something is itself equal to zero or it is equally $\frac{+1^{+1}}{+0^{+1}} \equiv +0$. Substituting this relationship into equation 3.24 we obtain

$$(3.25) \quad +0^{+1} \equiv +0^{+1} \times +0^{+1}$$

In general, we must accept that

$$(3.26) \quad +0^{+1} \equiv +0^{+2}$$

which contradicts theorem 3.4. It is possible to derive a contradiction out of the claim $+0^{+1} \equiv \frac{+0^{+1}}{+0^{+1}}$. This claim is refuted. □

3.6. $(1^{+1} / 0^{+1}) \neq (0^{+1} / 0^{+1})$.

Theorem 3.6 (Saitoh's logical fallacy). *In general, we must accept that*

$$(3.27) \quad \frac{+1^{+1}}{+0^{+1}} \neq \frac{+0^{+1}}{+0^{+1}}$$

Proof by modus inversus. **If** the premise

$$(3.28) \quad \underbrace{+1^{+1} = +0^{+1}}_{(Premise)}$$

is false, **then** the conclusion

$$(3.29) \quad \frac{+1^{+1}}{+0^{+1}} \equiv \frac{+0^{+1}}{+0^{+1}}$$

is false too. The premise is false. Dividing the premise by $+0^{+1}$, we obtain

$$(3.30) \quad \frac{+1^{+1}}{+0^{+1}} \equiv \frac{+0^{+1}}{+0^{+1}}$$

which is false too. In other words, we must accept in general that

$$(3.31) \quad \frac{+1^{+1}}{+0^{+1}} \neq \frac{+0^{+1}}{+0^{+1}}$$

and our conclusion is true. □

3.7. $+0^{+1} / +0^{+1} \neq +\infty^{+1}$. The evidence is increasing that $\frac{+0^{+1}}{+0^{+1}} \equiv \left(+0^{+1} \times \underbrace{\frac{+1^{+1}}{+0^{+1}}}_{\text{Something}} \right) \equiv +1^{+1}$. Repeatedly, we were fully capable to provide new (see theorem 3.5) evidence [3-5, 8] that $\frac{+0^{+1}}{+0^{+1}} \neq +0^{+1}$. However, why should it not be true that $\frac{+0^{+1}}{+0^{+1}} \equiv +\infty^{+1}$?

Theorem 3.7 (Zero divided by zero does not equal $+\infty$). *In general, we must accept that*

$$(3.32) \quad \frac{+0^{+1}}{+0^{+1}} \neq +\infty^{+1}$$

Proof by modus inversus. **If** the premise

$$(3.33) \quad \underbrace{+1 = +\infty}_{(Premise)}$$

is false, **then** the conclusion

$$(3.34) \quad \frac{+0^{+1}}{+0^{+1}} \equiv +\infty^{+1}$$

is false too. The premise is false. Rearranging equation 3.33 it is

$$(3.35) \quad +0^{+1} \times +1^{+1} \equiv +0^{+1} \times +\infty^{+1}$$

Rearranging equation 3.35, it is

$$(3.36) \quad \frac{+0^{+1}}{+0^{+1}} \equiv + \left(\frac{+0^{+1}}{+0^{+1}} \right) \times +\infty^{+1}$$

Our assumption is that $\frac{+0^{+1}}{+0^{+1}} \equiv +\infty^{+1}$. Substituting this into equation 3.36 we obtain

$$(3.37) \quad +\infty^{+1} \equiv + \left(+\infty^{+1} \times +\infty^{+1} \right) \equiv +\infty^{+2}$$

However, according to theorem 3.3, equation 3.37 is already proofed as incorrect. In other words, our assumption that $\frac{+0^{+1}}{+0^{+1}} \equiv +\infty^{+1}$ leads to an incorrect conclusion (equation 3.37). Therefore, we are justified to conclude that

$$(3.38) \quad \frac{+0^{+1}}{+0^{+1}} \neq +\infty^{+1}$$

and our conclusion is true. □

The theorem is valid for negative infinity too.

4. DISCUSSION

Indeed, there may exist different ways to reach the same goal and it appears to be that the more complex the problem, the more ways there are. However, classical logic and logically sound and consistent proof methods allow us to recognise and refute logically inconsistent scientific positions true to the motto '**The good into the pot, the bad into the crop**'. Furthermore, it can be learned from experience that there are indeed more ways of killing a dog than by hanging and to a greater or lesser extent it is the same with the refutation of incorrect theorem. Nonetheless, the evidence is overwhelming that $\frac{+0^{+1}}{+0^{+1}} \equiv +1^{+1}$. Neither is $\frac{+0^{+1}}{+0^{+1}} \equiv +0^{+1}$ nor is $\frac{+0^{+1}}{+0^{+1}} \equiv +\infty^{+1}$.

5. CONCLUSION

The hope is no longer justified that $\frac{+0^{+1}}{+0^{+1}} \equiv +0^{+1}$ or that $\frac{+0^{+1}}{+0^{+1}} \equiv +\infty^{+1}$.

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