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The division by zero.

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Abstract

It is claimed that a division by zero doesn't work, a division by zero is undefined, since the same appears to be not consistent with division by other numbers. A number multiplied by zero is zero, 0 times any number is 0. Thus some operations with zero are allowed. Any number divided by itself is one. Is zero a number or not? Does a number or a reality as such remains unchanged when it gets into contact with zero? Is zero divided by zero indeterminate or zero or one or undefined? Can we divide by zero? If not, why can't we divide by 0? Division by zero appears to be an operation for which one cannot find an answer. Does it make sense to divide by zero? Is dividing by zero really illegal? This publication will make the proof, that under certain conditions

the division by zero

is allowed.

Key words: Zero, One, General relativity, Einstein, Barukčić.

1. Background

Zero as the least non-negative, zero as neither the positive nor the negative, zero as the empty place indicator is a strange something. A division by zero is a division in mathematics where the divisor is zero. Such a division can be expressed as X / 0 = dividend / (divisor = 0). Equally it is claimed, that such an expression has no meaning. Thus, what is zero, who discovered zero?

There is evidence that the first appearance of zero as a number entered into Indian mathematics around 650AD (Colebrooke, 1817). However, we should note at this point that the Maya civilisation who lived in central America, used zero too. The work of the Indian mathematicians spread west to the Islamic countries as well as east to China. It appears to be that in 1247 the Chinese mathematician Ch'in Chiu-Shao used the symbol O for zero.

The problem which arises when we try to consider a **division by zero** if zero is the divisor are still unsolved. At first sight, the mathematical conception of zero appears to exclude the division by zero. Is a quiet different path towards the known concept of zero possible and allowed?

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2. Material and Methods

Zero is a number like any other but equally an natural process too. Zero has the ablility to change at least an unequal into an equal. Thus, there should be a connection to Einstein's basic field equation. Since even infinity has a lot of place within zero, Einstein's theory of general relativity, especially Einstein's basic field equation may explain the basic properties of zero. Thus, our starting point to solve the problem of zero is Einstein's basic field equation.

2.1. Einstein's field equation.

Recall, the below mathematical form of Einstein's field equation is for the -+++ metric sign convention. The -+++ metric sign convention is commonly used in general relativity. Einstein field equations were initially formulated in the context of a four-dimensional theory. However, Einstein field equations can be seen to hold in n dimensions too.

Einstein's basic field equation (EFE for the -+++ metric sign convention) defines zero.

Let

R_{ab} denote the Ricci tensor,

R denote the Ricci scalar,

g_{ab} denote the metric tensor,

T_{ab} denote the stress-energy tensor,

h denote Planck's constant, h ≈ **(6.626 0693 (11)) * 10 $^{-34}$ [J * s],**

π denote the mathematical constant π, also known as **Archimedes' constant.** The numerical value of π truncated to 50 decimal places is known to be about

π ≈ 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510,

c denote the speed of all electromagnetic radiation in a vacuum, the speed of light, where

$$c = 299\,792\,458 \ [m / s],$$

γ denote Newton's gravitational 'constant', where

$$\gamma \approx (6.6742 \pm 0.0010) * 10^{-11} \ [m^3 / (s^2 * kg)],$$

Einstein's field equation describes how a field or energy (or matter) and time changes space and vice versa. Einstein's basic field equation (EFE) is usually written in the form

$$(((4 * 2 * \pi * \gamma) * T_{ab}) / (c^4)) + ((R * g_{ab}) / 2)) = (R_{ab}). \qquad (1)$$

The stress-energy-momentum tensor is known to be the source of space-time curvature and describes more or less the density and flux of energy and momentum in space-time in Einstein's theory of gravitation. Philosophically, let the stress-energy-momentum tensor denote the **energy**.

The metric of space-time is determined by the matter and energy content of space-time. The Ricci scalar/metric tensor completely determines the curvature of space-time and defines such notions as *future, past,* distance, volume, angle. Philosophically, let the Ricci scalar/metric tensor denote the **time**.

The Ricci tensor, named after Gregorio Ricci-Curbastro, is a key term in the Einstein field equations and more or less a measure of *volume distortion*. Philosophically, let the Ricci tensor denote the **space**.

2.1.1. Properties of zero

2.1.1.1. Einstein's basic field equation

Zero is not only a number, zero is a natural process too, zero is defined by Einstein's basic field equation too.

Einstein's basic field equation (EFE for the $-+++$ metric sign convention) defines zero.

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Einstein's field equation describes how a field or energy (or matter) and time changes space and vice versa. Einstein's basic field equation (EFE) is usually written in the form

$$(((4 * 2 * \pi * \gamma) * T_{ab}) / (c^4)) + ((R * g_{ab}) / 2)) = (R_{ab}).$$

Then

$$0 = (((4 * 2 * \pi * \gamma) * T_{ab}) / (c^4)) + ((R * g_{ab}) / 2)) - (R_{ab}).$$

Proof.

$$(((4 * 2 * \pi * \gamma) * T_{ab}) / (c^4)) + ((R * g_{ab}) / 2)) = (R_{ab}) \quad (2)$$

$$(((4 * 2 * \pi * \gamma) * T_{ab}) / (c^4)) + ((R * g_{ab}) / 2)) - (R_{ab}) = (R_{ab}) - (R_{ab}) \quad (3)$$

$$0 = (((4 * 2 * \pi * \gamma) * T_{ab}) / (c^4)) + ((R * g_{ab}) / 2)) - (R_{ab}) \quad (4)$$

or

$$0 = (R_{ab}) - (((4 * 2 * \pi * \gamma) * T_{ab}) / (c^4)) - ((R * g_{ab}) / 2)) \quad (5)$$

Q. e. d.

If zero can be defined by Einstein`s basic field equation then it is equally true that there isn't something like an absolule and pure zero. Zero as such would in this case be something relative, every random variable would possess its own and relative zero. In so far, it appears reasonable to accept, that the world of the "true" and "pure" zero seems to be located in the region

$$(-h / (4 * \pi)) < 0 < (+h / (4 * \pi)).$$

2.1.1.2. The rules of precedence

Rule priority	Operation		Description	
1	Bracket operation	()	Bracket operation has to be performed before	(7)
2	Multiplication operation.	*	Multiplication operation has to be performed before	(8)
3	Division operation.	/	Division operation has to be performed before	(9)
4	Addition operation.	+	Addition operation has to be performed before	(10)
5	Subtraction operation.	-	Subtraction operation has to be performed before.	(11)
	...			

2.1.1.3. Basic rules

Adding, subtracting, multiplying and dividing two numbers are basic operations of algebra. Addition is the opposite of subtraction, as multiplication is the opposite of division. Any number multiplied by zero gives zero. The division by 0 is not impossible, division by zero is never allowed (Derbyshire 2004, p. 266). The division by 0 is only contradictory to a given set of rules and therefore not permitted. This set of rules are based on a special understanding of our world that is surrounding us and must not be true.

Recall. Addition and subtraction by zero.

$$+0+0 = +0 \quad (12)$$
$$+0+X = +X \quad (13)$$
$$+0-X = -X \quad (14)$$
$$+0-(-X) = +X \quad (15)$$
$$+0-(+X) = -X \quad (16)$$
$$+X-0 = +X \quad (17)$$
$$-X-0 = -X \quad (18)$$
$$+0-0 = +0 \quad (19)$$

Recall. Multiplication by zero.

$$0*0 = 0 \quad (20)$$
$$0^2 = 0 \quad (21)$$
$$0*0*0*...0 = 0 \quad (22)$$
$$0^n = 0 \quad (23)$$

$$1 \neq 0 \quad (24)$$
$$1 * 0 \neq 0 * 0 \quad (25)$$
$$0 = 0 \quad (26)$$

$$1 > 0 \quad (27)$$
$$1 * 0 > 0 * 0 \quad (28)$$
$$0 = 0 \quad (29)$$

$$1 < 0 \quad (30)$$
$$1 * 0 < 0 * 0 \quad (31)$$
$$0 = 0 \quad (32)$$

$$1 \leq 0 \quad (33)$$
$$1 * 0 \leq 0 * 0 \quad (34)$$
$$0 \leq 0 \quad (35)$$
$$0 = 0 \quad (36)$$

$$1 \geq 0 \quad (37)$$
$$1 * 0 \geq 0 * 0 \quad (38)$$
$$0 \geq 0 \quad (39)$$
$$0 = 0 \quad (40)$$

$$\text{limes}_{x \to 0}\left(\frac{\sin x}{x}\right) = 1. \quad (41)$$

$$0 * ! = 1. \quad (42)$$
$$\ln(1) = 0. \quad (43)$$

The multiplication of a number with zero has an other influence on a number then addition or subtraction. Zero as such has the power to change. An disequality (Eq. (24)) is changed into an equality (Eq. (26)) and sill and besides of all, the multiplication operation with zero is allowed. Only, an equality is not identical with and disequality both are different and not the same. Where does the difference vanish too? On the other hand, a strict inequality (Eq. (27), Eq. (30)) and a non-strict inequality (Eq. (33), Eq. (37)) is changed into an equality (Eq. (29), Eq. (32)) and Eq. (36), Eq. (40))) too. An inequality is different from an equality and vice versa. The number zero doesn’t care about this things.

Under certain circumstances, zero as a natural process is equalising differences. In zero, the distinguished are united and identical to each other. Where does the power of zero comes from? How can it change an inequality into an equality? On the other hand, Eq. (34) is stating more or less, that a division by zero doesn't lead to erroneous results. There are conditions, where a division by zero appears to be possible. Is the “negative zero” distinct from positive zero? A good set of rules should allow us to divide by zero. What could this rules be?

3. Results

Let us assume that a division by zero is allowed and that zero can be treated like any other number while respecting some other rules. Further, let us assume in that what follows, that the laws of classical logic and elementary algebra are valid. What are the consequences of this assumptions.

3.0. The definition of zero

The definition of zero.

Let

X denote something,

Anti X denote the opposite of X,

C = X + (Anti X),

0 denote zero that is related to X and Anti X,

Then

$$\mathbf{X * (Anti\ X) = 0.} \qquad (44)$$

Proof.

$$X = X \qquad (45)$$

$$X * (Anti\ X) = X * (Anti\ X) \qquad (46)$$

$$\text{Recall, } C = X + (Anti\ X). \qquad (46a)$$

$$\text{Thus, } Anti\ X = C - X. \qquad (46b)$$

$$X * (C - X) = X * (Anti\ X) \qquad (46c)$$

$$(\Delta X)^2 = (C * X) - (X * X) = X * (Anti\ X) \qquad (46d)$$

The laws of classical logic and elementary algebra are valid. According to classical logic something and its own opposite cannot exist at the same space-time or in other words. It is not possible, that X is at the same time X and equally Anti X, its own negation, its own opposite too. We obtain the next equation.

$$\mathbf{X * (Anti\ X) = 0} \qquad (47)$$

Q. e. d.

Recall, $(\Delta X)^2 = (C*X) - (X*X)$ is identified as the inner contradiction of X (Barukčić 2007d, p. 26). A definition of zero in accordance with classical logic and Einstein's relativity assumes, the zero is not an absolute zero, zero is a relative zero and is defined and determinated by the concrete relationship of X and Anti X. Zero, the empty, may define the emptiness and indeterminatedness but is equally determined by the relation of X and Anti X too. Zero as the "place" where X and Anti X withdraw is equally determinated by the same X and Anti X. This basic determinatedness of zero can be use to divide by zero. When we divide something by zero, we divide the same something at least by its own inner contradiction $(\Delta X)^2 = (C * X) - (X * X)$.

3.1. The definition of (X / 0)

The definition of (X / 0).

Let

X	denote something,
Anti X	denote the opposite of X,
0	denote zero that is related to X and Anti X.

Then

$$(X / 0) = (1/ (\text{Anti } X)). \quad (48)$$

Proof.

$$X = X \quad (49)$$

Let us assume that a division by zero is allowed and possible without any restrictions. We obtain the next relationship.

$$(X / 0) = (X / 0) \quad (50)$$

Recall, zero is determined by X and Anti X or (X * (Anti X)) = 0. Since the laws of classical logic are valid, we must accept the next equation too.

$$(X / 0) = (X / (X * (\text{Anti } X))) \quad (51)$$

$$(X / 0) = (X / X) *(1 / (\text{Anti } X)) \quad (52)$$

$$(X / 0) = (1) *(1 / (\text{Anti } X)) \quad (53)$$

$$(X / 0) = (1 / (\text{Anti } X)) \quad (54)$$

Q. e. d.

3.2. The definition of (0 / 0)

The expression (0 / 0) is known to be the most common example of an indeterminate forms too. The indeterminate form (0 / 0) is particularly common in calculus. Is it possible that an indeterminated is equally somehow the opposite of itself too, a determinated?

The definition of (0 / 0).

Let

X denote something,

Anti X denote the opposite of X,

0 denote zero that is related to X and Anti X.

Then

$$(0 / 0) = 1. \tag{54}$$

Proof.

$$X = X \tag{55}$$

Let us assume that a division by zero is allowed and possible without any restrictions. We obtain the next relationship.

$$(X / 0) = (X / 0)$$

Zero is determined by the basic relationship between X and (Anti X). We know, that (X * (Anti X)) = 0. We obtain the next relationship.

$$(X / 0) = (X / (X * (\text{Anti } X))) \tag{56}$$

$$(X / 0) = (X / X) *(1 / (\text{Anti } X)) \tag{57}$$

$$(X / 0) = (1) *(1 / (\text{Anti } X)) \tag{58}$$

$$(X / 0) = (1 / (\text{Anti } X)) \tag{59}$$

$$(X * (\text{Anti } X)) / 0 = 1 \tag{60}$$

Recall, zero is determined by X and Anti X or (X * (Anti X)) = 0. Since the laws of classical logic are valid, we must accept the next equation too.

$$(0 / 0) = 1 \tag{61}$$

Q. e. d.

3.3. The definition of (1 / 0)

It is claimed that the expression 1/0 is not an indeterminate form.

The definition of (1 / 0).

Let

X denote something,

Anti X denote the opposite of X,

0 denote zero that is related to X and Anti X.

Then

$$(1 / 0) = 1/(X*(\text{Anti } X)). \tag{62}$$

Proof.

$$X = X \tag{63}$$

Let us assume that a division by X is allowed and possible without any restrictions. We obtain the next relationship.

$$(X / X) = (X / X) \tag{64}$$

$$1 = (X / X) \tag{65}$$

$$(1 / (X * (\text{Anti } X))) = (X / X* (X * (\text{Anti } X))) \tag{66}$$

$$(1 / (X * (\text{Anti } X))) = (X / X)* (1 / (X * (\text{Anti } X))) \tag{67}$$

Zero is determined by the basic relationship between X and (Anti X). We know, that (X * (Anti X)) = 0. We obtain the next relationship.

$$(1 / 0) = (X / X)* (1 / (X * (\text{Anti } X))) \tag{68}$$

$$(1 / 0) = (1)* (1 / (X * (\text{Anti } X))) \tag{69}$$

Recall, zero is determined by X and Anti X or (X * (Anti X)) = 0. Since the laws of classical logic are valid, we must accept the next equation too.

$$(1 / 0) = 1 / (X * (\text{Anti } X)) \tag{70}$$

Q. e. d.

3.4. The proof $(1/0) \neq \infty$.

Sometimes it is claimed that $(1/0) = \infty$ and determines the infinity. Infinity isn't a real number, thus a value to (1/0) is not assigned, it is just said (1/0) is undefined. This contradicts our previous proof and at the end classical logic.

The proof $(1/0) \neq \infty$.

Let

X denote something,

Anti X denote the opposite of X,

0 denote zero that is related to X and Anti X.

Then

$$(1/0) \neq \infty. \tag{71}$$

Proof by contradiction.

$$X = X \tag{72}$$

Let us assume that a division by X is allowed and possible without any restrictions. We obtain the next relationship.

$$(X/X) = (X/X) \tag{72}$$

$$1 = (X/X) \tag{73}$$

$$(1/(X * (\text{Anti } X))) = (X/X * (X * (\text{Anti } X))) \tag{74}$$

$$(1/(X * (\text{Anti } X))) = (X/X) * (1/(X * (\text{Anti } X))) \tag{75}$$

Zero is determined by the basic relationship between X and (Anti X). We know, that $(X * (\text{Anti } X)) = 0$. We obtain the next relationship.

$$(1/0) = (X/X) * (1/(X * (\text{Anti } X))) \tag{76}$$

$$(1/0) = (1) * (1/(X * (\text{Anti } X))) \tag{77}$$

Recall, zero is determined by X and Anti X or $(X * (\text{Anti } X)) = 0$. Since the laws of classical logic are valid, we must accept the next equation too.

$$(1/0) = 1/(X * (\text{Anti } X)) \tag{78}$$

It is claimed that $(1 / 0) = \infty$ is true. Thus, let us accept this claim according to the rules of a proof by contradiction as correct. We obtain the next relationship.

$$(1 / 0) = \infty = 1 / (X * (\text{Anti } X)) \tag{79}$$

$$\infty = 1 / (X * (\text{Anti } X)) \tag{80}$$

According to the general contradiction law (Barukčić 2006e) it is equally true that $(X * (\text{Anti } X)) \leq (C^2 / 4)$. From this relationship, we obtain the next basic equation as $1 / (X * (\text{Anti } X)) \geq (4 / (C^2))$. This equation leads to the next relationship.

$$\infty = (4 / (C^2)) \tag{81}$$

$$\infty * C^2 = 4 \tag{82}$$

Set C = 1.

$$\infty * 1^2 = 4 \tag{83}$$

$$\infty = 4 \tag{84}$$

which is the contradiction.

Q. e. d.

Our assumption, that $(1 / 0) = \infty$ leads straight forward to a logical contradiction, if we accept the laws of classical logic as valid and if we accept, that a division by zero is allowed without any restriction. Thus, the opposite of $(1 / 0) = \infty$ must be accepted as true. Rules that allow and demand that $(1/0)=\infty$ should be reviewed.

The infinity as such may be placed between 0 and 1 but it appears equally not to be true that $(1/0) = \infty$.

3.5. The definition of (0 * X) = 1.

It is claimed that there is no X such that 0*X = 1, since 0*X = 0 for all X. On the other hand, equally it appears reasonable to accept that (1/0) does exist or is defined.

The definition of (X * 0 = 1).

Let

X denote something,

Anti X denote the opposite of X,

0 denote zero that is related to X and Anti X.

Then

$$X = 1/(X^*(\text{Anti } X)). \qquad (85)$$

Proof.

$$X * 0 = 1 \qquad (86)$$

Zero is determined by the basic relationship between X and (Anti X). We know, that (X * (Anti X)) = 0. We obtain the next relationship.

$$X * (X * (\text{Anti } X)) = 1 \qquad (87)$$

$$X = 1 / (X * (\text{Anti } X)) \qquad (88)$$

Q. e. d.

There is an X that way, that X * 0 = 1. Proof. X * 0 = 1. Set X = (1/(X*(Anti X))). Since, zero is defined by (X*(Anti X)) we obtain X * (X*(Anti X)) = 1 or equally (1/(X*(Anti X))) * (X*(Anti X)) = 1. This equation is true since the same yields 1 = 1. Q. e. d.

3.6. Faculty and Divculty

Christian Kramp introduced the notation *n*! for the factorial of a nonnegative integer *n* in the year 1808. The notation *n*! is pronounced as "n factorial". In mathematics, the factorial function can formally be defined by

$$n! = \left(\prod_{k=1}^{n} k\right) = 1*2*3*...*n \qquad \text{for all } n \in N. \qquad (89)$$

or

$$n! = \left(\prod_{k=0}^{n+1} k\right) / (n+1) \qquad \text{for all } n \in N. \qquad (90)$$

Recall,

$$0! = 1. \qquad (91)$$

$$1! = 1. \qquad (92)$$

$$n! = (n+1)! / (n+1) \qquad (93)$$

$$n! = n * (n-1)! \qquad (94)$$

$$n\ ! = 1*2*3*4*...*(n-1)*n = (n*1)! = n! \qquad (95)$$

Example

$$1*2*3*4*5*6 = 720 \qquad (96)$$

Divculty as the inverse operation of faculty which "does the reverse" of **faculty** that way that

Faculty * Divculty = 1.

Let

$$n\ ? = 1/(n!) = (1/(n*\Gamma(n))). \qquad (97)$$

Recall,

$$(n\ ?)*(n!) = 1 \qquad (98)$$

Example

$$n\ ? = ((((((1/2)/3)/4)/...)/(n-1))/n) = (n*1)\ ? \qquad (99)$$

$$(((((1/2)/3)/4)75)/6) = 1/720 = 1/(6!) = 0.001388... \qquad (100)$$

The factorial function can be defined for any nonnegative integer *n* using Euler's original formula for the Gamma function Γ(n). The Gamma function Γ(n) generalises the factorial function and is related to factorials in the following way.

$$n! = \Gamma(n+1) = n*\Gamma(n) = n*(n-1)! = n!*\Gamma(1). \qquad (101)$$

$$(n+1)!/(n+1) = n! = \Gamma(n+1) = n*\Gamma(n). \qquad (102)$$

$$(0+1)!/(0+1) = 0! = \Gamma(0+1) = 0*\Gamma(0). \qquad (103)$$

$$\Gamma(1/2) = (\pi)^{1/2} \qquad (104)$$

$$\Gamma(1) = 0! = 1 \qquad (105)$$

$$\Gamma(2) = 1! = 1 \qquad (106)$$

$$\Gamma(3) = 2! \qquad (107)$$

In so far, we can use the Gamma function Γ(n) instead of the factorial function in the following way:

$$n\,! = n\,! \tag{108}$$

$$n\,! = n * \Gamma(n) \tag{109}$$

Set Γ(...) = ! .

$$n\,! = n * ! \tag{110}$$

From this immediately follows.

$$n\,! = n * ! \tag{111}$$

$$n\,! / (\,!\,) = (n * \,!\,) / ! \tag{112}$$

$$n\,! / (\,!\,) = (n * \,!\,) / ! \tag{113}$$

$$n\,! / (\Gamma(n)) = n * (\,! / !\,) \tag{114}$$

$$n\,! / (\Gamma(n)) = n * (1) \tag{115}$$

$$n = n \tag{116}$$

$$n\,! = n * \Gamma(n) = n * (n - 1)\,! = n * !$$

It is worth to mention, that n ! = n * !. Meanwhile, mathematicians are distinguishing between the double factorial n!! and multifactorials such as $n!^{\,k}$. Triple factorial (*n*!!!) , superfactorial, hyperfactorial and so on are defined too.

What do we obtain when we perform the operation X * Γ (X) and divide this term by zero? The common perception is that (n! / 0) is still infinity.

The definition of (X!) / 0 .

Let

X	denote something,
Anti X	denote the opposite of X,
0	denote zero that is related to X and Anti X,
!	denote the factorial operation.

Then

$$(X!) / 0 = \Gamma(X)/(\text{Anti } X). \tag{117}$$

Proof.

$$X = X \tag{118}$$

Let us assume that a division by zero is allowed and possible without any restrictions. We obtain the next relationship.

$$(X / 0) = (X / 0) \tag{119}$$

Recall, zero is determined by X and Anti X or (X * (Anti X)) = 0. Since the laws of classical logic are valid, we must accept the next equation too.

$$(X / 0) = (X / (X * (\text{Anti } X))) \tag{120}$$

$$(X / 0) = (X / X) *(1 / (\text{Anti } X)) \tag{121}$$

$$(X / 0) = (1) *(1 / (\text{Anti } X)) \tag{122}$$

$$(X / 0) = (1 / (\text{Anti } X)) \tag{123}$$

$$(X * \Gamma (X)) / 0 = (1 * \Gamma (X))/ (\text{Anti } X) \tag{124}$$

Recall, $(X !) = \Gamma(X +1)=(X*\Gamma(X))$.

$$(X !)/0 = \Gamma(X +1)/ 0=(X*\Gamma(X)) / 0 = \Gamma (X) / (\text{Anti } X). \tag{125}$$

$$(X !) / 0 = \Gamma(X) / (\text{Anti } X) \tag{126}$$

Q. e. d.

3.7. The division by zero and proofs

There are lot of "proofs" claiming to prove that the division by zero is not possible and obviously not true. All of these "proofs" contain some error. The most common trick is not to respect the rules of precedence, to divide an equation by something else that equals zero and not by zero itself. Something the equals zero is not zero it self, it equals only zero. If a "proof" divides by something else that equals zero and not by zero itself, this "proof" can "prove" anything it wants to, including false statements. Such proofs contain some error, and are therefore not real proofs

Claim.

$$2 = 0. \tag{127}$$

Proof.

$$a = b \tag{128}$$

$$a^2 = b^2 \tag{129}$$

$$a^2 - b^2 = 0 \tag{130}$$

$$(a-b)*(a+b) = 0 \tag{131}$$

$$((a-b)*(a+b))/(a-b) = 0/(a-b) \tag{132}$$

$$\text{Set } a = 1. \text{ Set } b = 1 \tag{133}$$

$$(1*(a+b)) = 0 \tag{134}$$

$$(1*(1+1)) = 0 \tag{135}$$

$$(1*(2)) = 0 \tag{136}$$

$$2 = 0 \tag{137}$$

Q. e. d.

It is of course not true, that 2 = 0. It is claimed, that the division by zero is responsible for this error which is not true. The error in this proof is the misuses of the rules of precedence and of the fact that it is not divided by 0 but by a – b. It is assumed that a – b = 0 but zero as such is not used. The variables a and b can change. If the difference between a and b equals 0, then 0 as such must be used and not difference between a – b, since this difference could take other values too. It must be assured that zero is zero. We cannot accept saying we are dividing by zero while the same is not assured. Further, the division by zero assumes that the rules of precedence are respected strictly. Thus let us repeat the same proof once again while respecting the properties of zero and thus the rules and laws accompanied with zero.

Claim.

$$2 = 0. \tag{138}$$

Proof.

$$a = b \tag{139}$$

$$a^2 = b^2 \tag{140}$$

$$a^2 - b^2 = 0 \tag{141}$$

$$(a-b)*(a+b) = 0 \tag{142}$$

$$((a-b)*(a+b))/0 = 0/0 \quad (143)$$

Set a = 1. Set b = 1

$$((1-1)*(a+b))/0 = 0/0 \quad (144)$$

Bracket operations must be performed before multiplication.

$$(0*(a+b)) /0 = 0/0 \quad (145)$$

$$(0*(1+1)) /0 = 0/0 \quad (146)$$

Multiplication before division!

$$(0*(2)) /0 = 0/0 \quad (147)$$

At the end, division by zero.

$$0 / 0 = 0/0 \quad (148)$$

$$1 = 1 \quad (149)$$

Q. e. d.

Claim.

$$2 = 1 \quad (150)$$

Proof.

$$a = b \quad (151)$$

$$a*a = (b*a) \quad (152)$$

$$a^2 - b^2 = (a*b) - b^2 \quad (153)$$

$$(a-b)*(a+b) = b*(a - b) \quad (154)$$

$$((a-b)*(a+b)) / (a-b) = (b*(a - b))/ (a-b) \quad (155)$$

$$(a+b) = b \quad (156)$$

Recall, a = b. (157)

$$(b+b) = b \quad (158)$$

$$2*b = b \quad (159)$$

$$2 = 1 \quad (160)$$

Q. e. d.

The division by zero yields once again a logical contradiction. It is of course not true that 2 = 1 and it is claimed that the division by zero is responsible for the result above. Only we have not divided by 0, we have divided by a – b. The term a - b can be equal to zero but must not be. If we divide by zero, then we must use zero as such and not the term a – b. On the other hand, the rules of precedence must be respected as such. This is not the case at all. Thus, let us repeat the proof above once again while respecting the rules.

Claim.

$$1 = 1 \qquad (161)$$

Proof.

$$a = b \qquad (162)$$

$$a^2 = (a*b) \qquad (163)$$

$$a^2 - b^2 = (a*b) - b^2 \qquad (164)$$

$$(a-b)*(a+b) = b*(a - b) \qquad (165)$$

$$((a-b)*(a+b)) / (a-b) = (b*(a - b))/(a-b) \qquad (166)$$

Recall, a = b.

$$((a-a)*(b+b)) / (a-a) = (b*(a - a))/(a-a) \qquad (167)$$

Bracket operation must before multiplication and division.

$$((0)*(2*b)) / (0) = (b*(0))/(0) \qquad (168)$$

Multiplication must before division.

$$(0 / 0) = (0 / 0) \qquad (169)$$

At the end, division by zero.

$$1 = 1 \qquad (170)$$

Q. e. d.

4. Discussion

At first glance, the division by zero is defined, operations involving the division by zero are possible and allowed. The division by zero can lead to paradoxes if the rules on which the same division is based are not respected carefully.

Although division by zero is said not to be defined there are situations in which division by zero in fact can be considered as defined. The rules, that allow the division by zero should be developed in more detail.

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