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Conditio per quam

Research article

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Abstract

Background:

Despite the fact that orthodox classical logic is able to deal with the most basic and most-simple laws of objective reality, 'non-classical' logic or probability theory have been fruitfully applied in areas as diverse as philosophy, (quantum) physics and mathematics et cetera too.

Methods:

At this stage, it seems fair to say that this publication discusses the unification of classical logic and probability theory and attempts to provide a clarification of the material implication (conditio per quam) relationship.

Results:

In contrast to classical logic which offers a qualitative (structural) view on objective reality, 'non-classical' logic or probability theory is quantitative (numerical) in nature. After all, both are concerned and used to describe the conditio per quam relationship completely.

Conclusion:

By integrating the perspectives of qualitative logic and numerical probability theory, the material implication or conditio per quam is reformulated and able to offer highly expressive accounts of inference.

Keywords: Material implication; Conditio per quam; Cause; Effect; Causation

1. Introduction

In spite of the discussed limitations (see Łukasiewicz, 1920, Post, 1921) of the two-valued or bivalent (see DeVidi and Solomon, 1999) classical (see Sandqvist, 2009) logic and the need to treat the probability of an event as the truth value of a many-valued or non-classical logic, there is sufficient historical evidence that Boolean algebra (see Boole, 1854) dominated treatment of 'material implication' (see Fulda, 1989, Gerhard, 1841, Lewis, 1917b, Russell, 1906) stems more or less from the two-valued propositional framework of Frege (see Frege, 1879) and Whitehead and Russell (see Russell and Whitehead, 1910). However, the traceable history of documented discussions by several authors of the relationship conditio per quam or the notion 'material implication'dates back (see Sedley, 1977) more than 2000 years. Famously, a small group of early Hellenistic philosophers, including Diodorus Cronus (see Benson, 1949) and Philo the Logician, a disciple of Diodorus Cronus, made essential contributions to the theory of material implication. In antiquity, Philo the Dialectician (fl. 300 BCE), an outstanding philosopher of the Megarian (Dialectical) school, introduced that **a material implication is true exactly when it is not the case that** the conditioned (B_t) is false and the condition (A_t) is true (Sextus Empiricus, Adv. Math. viii, Section 113). A small chronological list [1] of the members of the Dialectical school can be found in secondary literature. There are many different things one can say about the former (see Lewis, 1917a, 1912, Russell and Whitehead, 1910) and contemporary (see Barukčić, 1989) concept of material implication. Especially the theory of material implication as developed by Russel and other (see Dale, 1974, Farrell, 1979, Fulda, 1989) logicians has met with a considerable (see Quine, 1940, p. 31) degree of objections (see also Brandom, 1981, Mansur, 2005, Nelson, 1966, Wiener, 1916). The starting point of Lewis logic is the claim that '⋯ a false proposition implies any proposition ⋯ ' (see Lewis, 1912) or in general, from contradictory premises, anything follows. Lewis fails to do justice to the nature of material implication and advocates (see Lewis, 1912) the validity of the ex contradictione quodlibet (ECQ) principle which is meanwhile refuted (see Barukčić, 2017c, 2019a, 2020d, Barukčić and Ufuoma, 2020, Barukčić, Ilija, 2019). Material implication is very tolerant on the case, if the condition (A_t) itself is not given. Under these conditions, the conditioned (B_t) can occur with the probability $p(c_t)$ but need not to occur with the probability $p(d_t)$ (see Fig. 1). Importantly, this fact has inspired to many authors to commit a logical

Table 1. Sufficient condition.

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	**+0**	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

fallacy that from nothing or the non-existence of something (condition A_t is not given or will not occur with the probability $p(\underline{A}_t)$) anything might follow.

"There are many philosophers to whom you can not mention the name 'Russell,'without evoking such comments as, 'H i s logic is purely artificial, for it is nonsense to suppose that a false proposition implies any proposition, or that any proposition implies any true proposition,' or, 'Who could ever reasonably maintain that, 'The moon is made of green cheese,' implies, 'Caesar died in his bed?' "

(see also Wiener, 1916)

Some paradoxes of material implication are due to an inconsistent and invalidate approach to the nature of material implication and the miss-match of the rule of modus ponens (see McGee, 1985) with material implication. In general, **conditio per quam** or 'material implication' (see Gerhard, 1841) is about the relationship between events of the form "**if** A_t, **then** B_t"at the same (period of) time t / Bernoulli trial t (see Uspensky, 1937). However, what is it for a conditioned B_t to be a determined

[1] Bobzien, Susanne, "Dialectical School", The Stanford Encyclopedia of Philosophy (Spring 2019 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/spr2019/entries/dialectical-school/.

by a condition A_t in order to prevent paradoxes (see also Brandom, 1981, Mansur, 2005) of material implication at the same (period of) time t / Bernoulli trial t (see Uspensky, 1937) might be illustrated by table 2, Bernoulli trial 2.

Table 2. Conditio per quam (sufficient condition).

Bernoulli trial t	A_t	B_t	$A_t \rightarrow B_t \equiv (\underline{A}_t \vee B_t)$
1	1	1	1
2	1	0	0
3	0	1	1
4	0	0	1
...	...	...	...

Nonetheless, material implication (see also Lewis, 1917a, Wiener, 1916) or conditio per quam relationship can be visualised by a Venn diagram as popularised 1881 by John Venn (1834–1923), an English logician, mathematician and philosopher, in his book Symbolic Logic (see also Venn, 1881, pp. 100-125), a specification of Euler diagrams (see also Euler, 1768).

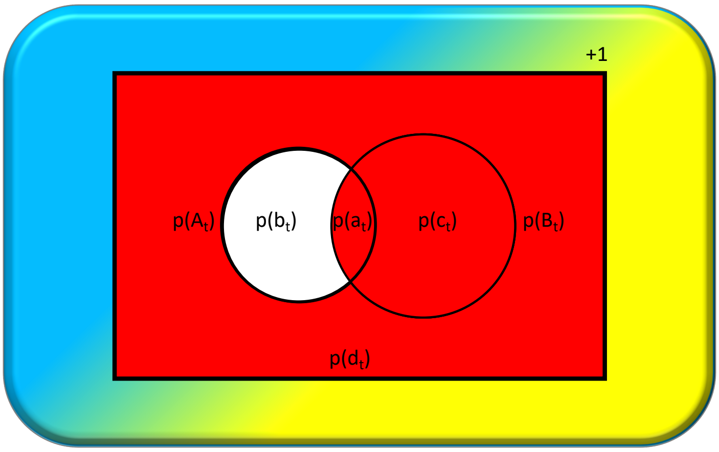

Figure 1. Material implication and Venn diagram

However, there is surprisingly little or none agreement about what the right form of a material implication might be while expressing the same relationship by the tools of statistics and probability theory. In what follows, we will represent probability theory dominated treatment of material implication. Additionally, our aim in this article is to provide a brief characterisation of fundamental features of material implication.

2. Material and methods

Scientific knowledge and objective reality are more than interrelated. Objective reality is the foundation of any scientific knowledge. Our human experience teaches us however that seen by light, grey is never merely simply grey, and looked at from different angles, many paths may lead to climb up a certain mountain. In general, it is appropriate to ensure as much as possible a broader consideration of a research question and to take into account the different facets and viewpoints of an issue investigated in order to reach a goal.

2.1. Methods

Definitions should help us to provide and assure a systematic approach to a mathematical formulation of the relationship of a necessary condition. It also goes without the need of further saying that a definition must be logically consistent and correct.

2.1.1. Random variables

Let a **random variable**(Gosset, 1914) X denote something like a function defined on a probability space, which itself maps from the sample space(Neyman and Pearson, 1933) to the real numbers.

2.1.2. The Expectation of a Random Variable

Definition 2.1 (**The First Moment Expectation of a Random Variable**). *Summaries of an entire distribution of a random variable(see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 22) X, such as the expected value, or average value, are useful in order to identify where X is expected to be without describing the entire distribution. For practical and other reasons, we shall limit ourselves here to discrete random variables, while the basic properties of the expectation value of a random variable X will not be investigated. Thus far, let X be a discrete random variable with the probability p(X). The first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of X, denoted by E(X), is a number defined as follows:*

$$E(X) \equiv p(X) \times X \tag{1}$$

The first moment expectation value squared of a random variable X follows as

$$\begin{aligned} E(X)^2 &\equiv p(X) \times X \times p(X) \times X \\ &\equiv p(X) \times p(X) \times X \times X \\ &\equiv (p(X) \times X)^2 \\ &\equiv E(X) \times E(X) \end{aligned} \tag{2}$$

The ongoing progress with artificial intelligence has the potential to transform human society far beyond any imaginable border of human recognition and can help even to solve problems that otherwise would not be tractable. No wonder, scientist and systems are confronted with large volumes of data (big data) of various natures and from different sources. The use of tensor technology can simplify and accelerate Big data analysis. In other words, let $X_{kl\mu\nu\ldots}$ denote an n-th index co-variant tensor with the probability $p(X_{kl\mu\nu\ldots})$. The first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of $X_{kl\mu\nu\ldots}$, denoted by $E(X_{kl\mu\nu\ldots})$, is a number defined as follows:

$$E\left(X_{kl\mu\nu\ldots}\right) \equiv p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \equiv p\left(X_{kl\mu\nu\ldots}\right) \cap X_{kl\mu\nu\ldots} \tag{3}$$

while $\times$ or $\cap$ might denote the commutative multiplications of tensors. The first moment expectation value squared of a random variable X follows as

$$\begin{aligned} {}^{2}E\left(X_{kl\mu\nu\ldots}\right) &\equiv p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \times p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \\ &\equiv p\left(X_{kl\mu\nu\ldots}\right) \times p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \times X_{kl\mu\nu\ldots} \\ &\equiv {}^{2}\left(p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots}\right) \\ &\equiv E\left(X_{kl\mu\nu\ldots}\right) \times E\left(X_{kl\mu\nu\ldots}\right) \end{aligned} \tag{4}$$

Definition 2.2 (**The Second Moment Expectation of a Random Variable**). *The second(see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42) moment expectation value (or more or less arithmetic mean) of a (large) number of independent realizations of a random variable X follows as:*

$$\begin{aligned} E\left(X^{2}\right) &\equiv p(X) \times X^{2} \\ &\equiv (p(X) \times X) \times X \\ &\equiv E(X) \times X \\ &\equiv X \times E(X) \end{aligned} \tag{5}$$

From the point of view of tensor algebra it is

$$\begin{aligned} E\left({}^{2}X_{kl\mu\nu\ldots}\right) &\equiv p\left(X_{kl\mu\nu\ldots}\right) \times {}^{2}X_{kl\mu\nu\ldots} \\ &\equiv \left(p\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \\ &\equiv E\left(X_{kl\mu\nu\ldots}\right) \times X_{kl\mu\nu\ldots} \\ &\equiv X_{kl\mu\nu\ldots} \times E\left(X_{kl\mu\nu\ldots}\right) \end{aligned} \tag{6}$$

Definition 2.3 (**The n-th Moment Expectation of a Random Variable**). *The n-th(see Barukčić, 2020a, 2021c) moment expectation value of a (large) number of independent realizations of a random variable X follows as:*

$$\begin{aligned} E\left(X^{n}\right) &\equiv p(X) \times X^{n} \\ &\equiv (p(X) \times X) \times X^{n\text{-}1} \\ &\equiv E(X) \times X^{n\text{-}1} \end{aligned} \tag{7}$$

2.1.3. Probability of a Random Variable

The probability p(X) of a random variable X follows as (see equation 1)

$$
\begin{aligned}
p(X) &\equiv \frac{X \times p(X)}{X} \equiv \frac{E(X)}{X} \\
&\equiv \frac{X \times X \times p(X)}{X \times X} \equiv \frac{E\left(X^2\right)}{X^2} \\
&\equiv \frac{E(X) \times E(X)}{E(X) \times X} \equiv \frac{E(X)^2}{E\left(X^2\right)} \\
&\equiv \Psi(X) \times \Psi^{*}(X)
\end{aligned}
\tag{8}
$$

where $\Psi(X)$ is the wave-function of X, $\Psi^{*}(X)$ is the complex conjugate wave-function of X. From the point of view of tensor algebra, we obtain

$$
\begin{aligned}
p\left(X_{\mathrm{kl}\mu\nu\ldots}\right) &\equiv \frac{X_{\mathrm{kl}\mu\nu\ldots} \times p\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{X_{\mathrm{kl}\mu\nu\ldots}} \equiv \frac{E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{X_{\mathrm{kl}\mu\nu\ldots}} \\
&\equiv \frac{X_{\mathrm{kl}\mu\nu\ldots} \times X_{\mathrm{kl}\mu\nu\ldots} \times p\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{X_{\mathrm{kl}\mu\nu\ldots} \times X_{\mathrm{kl}\mu\nu\ldots}} \equiv \frac{E\left({}^{2}X_{\mathrm{kl}\mu\nu\ldots}\right)}{{}^{2}X_{\mathrm{kl}\mu\nu\ldots}} \\
&\equiv \frac{E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times X_{\mathrm{kl}\mu\nu\ldots}} \equiv \frac{{}^{2}E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)}{E\left({}^{2}X_{\mathrm{kl}\mu\nu\ldots}\right)} \\
&\equiv \Psi\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times \Psi^{*}\left(X_{\mathrm{kl}\mu\nu\ldots}\right)
\end{aligned}
\tag{9}
$$

where $\Psi\left(X_{\mathrm{kl}\mu\nu\ldots}\right)$ is the wave-function tensor of $\mathrm{X}_{\mathrm{kl}\mu\nu\ldots}$, $\Psi^{*}\left(X_{\mathrm{kl}\mu\nu\ldots}\right)$ is the complex conjugate wave-function tensor of $\mathrm{X}_{\mathrm{kl}\mu\nu\ldots}$.

2.1.4. Variance and Co-variance of a Random Variable

Definition 2.4 (The Variance of a Random Variable). *Johann Carl Friedrich Gauß (1777-1855) introduced the normal distribution and the error of mean squared in his 1809 monograph(see Gauß, Carl Friedrich, 1809). In the following, Karl Pearson (1857-1936) coined the term "standard deviation" in 1893. Pearson is writing: "Then σ will be termed its standard-deviation (error of mean square)."(see Pearson, 1894, p. 80). Finally, the term variance was introduced by Sir Ronald Aylmer Fisher (1890-1962) in the year 1918.*

> *"The ... deviations of a ... measurement from its mean ... may be ... measured by the standard deviation corresponding to the square root of the mean square error ... It is ... desirable **in analysing the causes** ... to deal with the square of the standard deviation as the measure of variability. We shall term this quantity the Variance... "*
>
> *(see Fisher, Ronald Aylmer, 1919, p. 399)*

The deviation of a random variable X from its population mean or sample mean E(X) has a central role in statistics and is one important measure of dispersion. The variance $\sigma(X)^2$ (see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42), the second central moment of a distribution, is the expectation value of the squared deviation of a random variable X from its own expectation value E(X) and is determined in general as (see equation 5):

$$\begin{aligned}\sigma(X)^2 &\equiv E\left(X^2\right) - E(X)^2 \\ &\equiv (X \times E(X)) - E(X)^2 \\ &\equiv E(X) \times (X - E(X)) \\ &\equiv E(X) \times E(\underline{X})\end{aligned} \tag{10}$$

while $E(\underline{X}) \equiv X - E(X)$. From the point of view of tensor algebra, it is

$$\begin{aligned}{}^2\sigma\left(X_{\mathrm{kl}\mu\nu\ldots}\right) &\equiv E\left({}^2X_{\mathrm{kl}\mu\nu\ldots}\right) - {}^2E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \\ &\equiv \left(X_{\mathrm{kl}\mu\nu\ldots} \times E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)\right) - {}^2E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \\ &\equiv E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times \left(X_{\mathrm{kl}\mu\nu\ldots} - E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)\right) \\ &\equiv E\left(X_{\mathrm{kl}\mu\nu\ldots}\right) \times E\left(\underline{X}_{\mathrm{kl}\mu\nu\ldots}\right)\end{aligned} \tag{11}$$

while $E\left(\underline{X}_{\mathrm{kl}\mu\nu\ldots}\right) \equiv X_{\mathrm{kl}\mu\nu\ldots} - E\left(X_{\mathrm{kl}\mu\nu\ldots}\right)$. As demonstrated by equation 11, variance depends not just on the expectation value of what has actually been observed $E\left(\left(X_{\mathrm{kl}\mu\nu\ldots}\right)\right)$, but also on the expectation value that could have been observed but were not $\left(E\left(\underline{X}_{\mathrm{kl}\mu\nu\ldots}\right)\right)$. There are circumstances in quantum mechanics where this fact is called the local hidden variable. Even if his might strike us as peculiar, variance [2] is primarily a mathematical method which is of use in order to evaluate specific hypotheses in the light of some empirical facts. However, as a mathematical tool or method, variance is also a scientific description of a certain part of objective reality too. In this context, as a general mathematical principle, one fundamental meaning of variance is to provide a logically consistent link between something and its own other, between X and anti X.

> "The variance in this sense is a measure of the inner contradictions of a random variable, of changes, of struggle within this random variable itself, or the greater $\sigma(X)^2$ of a random variable, the greater the inner contradictions of this random variable. "
>
> (see Barukčić, 2006a, p. 57)

All things considered, we can safely say that, on the whole, **the variance is a mathematical description of the philosophical notion of the inner contradiction of a random variable X** (see Hegel, Georg Wilhelm Friedrich, 1812, 1813, 1816) . Based on equation 10, it is

$$E\left(X^2\right) \equiv E(X)^2 + \sigma(X)^2 \tag{12}$$

or

$$\frac{E(X)^2}{E(X^2)} + \frac{\sigma(X)^2}{E(X^2)} \equiv p(X) + \frac{\sigma(X)^2}{E(X^2)} \equiv +1 \tag{13}$$

In other words, the variance (see Barukčić, 2006b) of a random variable is a determining part of the probability of a random variable. The wave function Ψ follows in general, as

$$\begin{aligned}
\Psi(X) &\equiv \frac{1}{\Psi^*(X)} - \frac{\sigma(X)^2}{(\Psi^*(X) \times E(X^2))} \\
&\equiv \frac{\left(E(X^2) - \sigma(X)^2\right)}{(\Psi^*(X) \times E(X^2))} \\
&\equiv \frac{1}{(\Psi^*(X) \times E(X^2))} \times \left(E\left(X^2\right) - \sigma(X)^2\right) \\
&\equiv \frac{1}{(\Psi^*(X) \times E(X^2))} \times E(X)^2 \\
&\equiv \frac{1}{\Psi^*(X)} \times \frac{E(X)^2}{E(X^2)} \\
&\equiv \frac{1}{\Psi^*(X) \times X} \times E(X)
\end{aligned} \tag{14}$$

The wave function (see Born, 1926) of a quantum-mechanical system is a central determining part of the Schrödinger wave equation (see Schrödinger, Erwin Rudolf Josef Alexander, 1926, 1929, 1952).

[2] Romeijn, Jan-Willem, "Philosophy of Statistics", The Stanford Encyclopedia of Philosophy (Spring 2022 Edition), Edward N. Zalta (ed.), forthcoming URL = https://plato.stanford.edu/archives/spr2022/entries/statistics/.

Definition 2.5 (**The First Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**)**.** *In general, let $E(\underline{X})$ be defined as*

$$E(\underline{X}) \equiv X - E(X) \equiv X - (X \times p(X)) \tag{15}$$

and denote an expectation value of a (discrete) random variable anti X with the probability

$$p(\underline{X}) \equiv 1 - p(X) \tag{16}$$

The first moment expectation value (see Huygens and van Schooten, 1657, Kolmogorov, Andreĭ Nikolaevich, 1950, LaPlace, 1812, Whitworth, 1901) of anti X, denoted as E($\underline{X}$), is a number defined as follows:

$$E(\underline{X}) \equiv X - (X \times p(X)) \equiv X \times (1 - p(X)) \equiv X \times p(\underline{X}) \tag{17}$$

The first moment expectation value squared of a random variable anti X follows as

$$\begin{aligned} E(\underline{X})^2 &\equiv p(\underline{X}) \times X \times p(\underline{X}) \times X \\ &\equiv p(\underline{X}) \times p(\underline{X}) \times X \times X \\ &\equiv (p(\underline{X}) \times X)^2 \\ &\equiv E(\underline{X}) \times E(\underline{X}) \end{aligned} \tag{18}$$

Definition 2.6 (**The Second Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**)**.** *The second(see Kolmogorov, Andreĭ Nikolaevich, 1950, p. 42) moment expectation value (or more or less arithmetic mean) of a (large) number of independent realizations of a random variable anti X follows as:*

$$\begin{aligned} E(\underline{X}^2) &\equiv p(\underline{X}) \times X^2 \\ &\equiv (p(\underline{X}) \times X) \times X \\ &\equiv E(\underline{X}) \times X \\ &\equiv X \times E(\underline{X}) \end{aligned} \tag{19}$$

Definition 2.7 (**The n-th Moment Expectation of a Random Variable of $\underline{X}$ (anti X)**)**.** *The n-th(see Barukčić, 2020a, 2021c) moment expectation value of a (large) number of independent realizations of a random variable anti X follows as:*

$$\begin{aligned} E(\underline{X}^n) &\equiv p(\underline{X}) \times X^n \\ &\equiv (p(\underline{X}) \times X) \times X^{n\text{-}1} \\ &\equiv E(\underline{X}) \times X^{n\text{-}1} \end{aligned} \tag{20}$$

Definition 2.8 (**The Co-Variance of a Random Variable**)**.** *Sir Ronald Aylmer Fisher (1890 -1962) introduced the term covariance (see Bailey, 1931) in the year 1930 in his book as follows:*

> *"It is obvious too that where a considerable fraction of the variance is contributed by chance causes, the variance of any group of individuals will be inflated in comparison with the covariances between related groups ... "*
>
> *(see Fisher, Ronald Aylmer, 1930, p. 195)*

In general, the co-variance is defined as given by equation 21.

$$\sigma(X,Y) \equiv E(X,Y) - (E(X) \times E(Y)) \tag{21}$$

From the point of view of tensor algebra, it is

$$\sigma\left(X_{kl\mu\nu\ldots}, Y_{kl\mu\nu\ldots}\right) \equiv E\left(X_{kl\mu\nu\ldots}, Y_{kl\mu\nu\ldots}\right) - \left(E\left(X_{kl\mu\nu\ldots}\right) \times E\left(Y_{kl\mu\nu\ldots}\right)\right) \tag{22}$$

2.1.5. Bernoulli distribution

A single event distribution is more or less a discrete probability distribution of any random variable X which takes a certain (observer independent) single value X_t at a **Bernoulli trial** (Uspensky, 1937, p. 45) (period of time) t with the probability p(X_t). The same random variable X takes a certain single anti value $\underline{X}_t$ at a Bernoulli trial (period of time) t with the probability 1-p(X_t). There are conditions in nature where a random variable X can take only the values either +0 or +1 (see Birnbaum, 1961). Under these conditions, the random variable X takes the value 1 with probability p(X_t = +1) and the value 0 with probability $q(X_t = +0) = 1 - p(X_t = +1)$ while the single event distribution passes over into the **Bernoulli distribution**, named after Swiss mathematician Jacob Bernoulli (Bernoulli, 1713). Less formally, many times, the Bernoulli distribution is represented by a (possibly not biased) coin toss where 1 and 0 would represent 'heads'and 'tails'(or vice versa), respectively. However, the relationship between random variables (Gosset, 1914) can be investigated by many (Gosset, 1908) methods, including the tools of probability theory, too.

Definition 2.9 (Two by two table of single event random variables).

The two by two or contingency table which has been introduced by Karl Pearson (Pearson, 1904b) in 1904 harbours still a large variety of topics and debates. Central to this is the problem to apply the laws of classical logic on data sets, which concerns the justification of inferences which extrapolate from sample data to general facts. Nevertheless, a contingency table is still an appropriate theoretical model too for studying the relationships between random variables, including *Bernoulli (Bernoulli, 1713) (i.e. +0/+1) distributed random variables* existing or occurring at the same *Bernoulli trial* (Uspensky, 1937) (period of time) t.

In this context, let a random variable A at the *Bernoulli trial* (Uspensky, 1937) (period of time) t, denoted by A_t, indicate a risk factor, a condition, a cause et cetera and occur or exist with the probability p(A_t) at the *Bernoulli trial* (Uspensky, 1937) (period of time) t. Let E(A_t) denote the expectation value of A_t. In general it is

$$p(A_t) \equiv p(a_t) + p(b_t) \tag{23}$$

The expectation value E(A_t) follows as

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv A_t \times (p(a_t) + p(b_t)) \\ &\equiv (A_t \times p(a_t)) + (A_t \times p(b_t)) \\ &\equiv E(a_t) + E(b_t) \end{aligned} \tag{24}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E(A_t) &\equiv A_t \times p(A_t) \\ &\equiv (+0+1) \times p(A_t) \\ &\equiv p(A_t) \\ &\equiv p(a_t) + p(b_t) \end{aligned} \tag{25}$$

Furthermore, it is

$$p\left(\underline{A}_{\mathrm{t}}\right) \equiv p\left(c_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right) \equiv \left(1-p\left(A_{\mathrm{t}}\right)\right) \tag{26}$$

The expectation value E($\underline{\mathrm{A}}_{\mathrm{t}}$) is given as

$$\begin{aligned}
E\left(\underline{A}_{\mathrm{t}}\right) &\equiv A_{\mathrm{t}} \times \left(1-p\left(A_{\mathrm{t}}\right)\right)\\
&\equiv A_{\mathrm{t}} \times \left(p\left(c_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right)\right)\\
&\equiv \left(A_{\mathrm{t}} \times p\left(c_{\mathrm{t}}\right)\right)+\left(A_{\mathrm{t}} \times p\left(d_{\mathrm{t}}\right)\right)\\
&\equiv E\left(c_{\mathrm{t}}\right)+E\left(d_{\mathrm{t}}\right)
\end{aligned} \tag{27}$$

Under conditions of +0/+1 distributed Bernoulli random variables we obtain

$$\begin{aligned}
E\left(\underline{A}_{\mathrm{t}}\right) &\equiv A_{\mathrm{t}} \times \left(1-p\left(A_{\mathrm{t}}\right)\right)\\
&\equiv (+0+1) \times \left(1-p\left(A_{\mathrm{t}}\right)\right)\\
&\equiv \left(1-p\left(A_{\mathrm{t}}\right)\right)\\
&\equiv p\left(c_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right)
\end{aligned} \tag{28}$$

Let a random variable B at the *Bernoulli trial* (Uspensky, 1937) (period of time) t, denoted by B_{t}, indicate an outcome, a conditioned, an effect et cetera and occur or exist with the probability p(B_{t}) at the *Bernoulli trial* (Uspensky, 1937) (period of time) t. Let E(B_{t}) denote the expectation value of B_{t}. In general it is

$$p\left(B_{\mathrm{t}}\right) \equiv p\left(a_{\mathrm{t}}\right)+p\left(c_{\mathrm{t}}\right) \tag{29}$$

The expectation value E(B_{t}) is given by the equation

$$\begin{aligned}
E\left(B_{\mathrm{t}}\right) &\equiv B_{\mathrm{t}} \times p\left(B_{\mathrm{t}}\right)\\
&\equiv B_{\mathrm{t}} \times \left(p\left(a_{\mathrm{t}}\right)+p\left(c_{\mathrm{t}}\right)\right)\\
&\equiv \left(B_{\mathrm{t}} \times p\left(a_{\mathrm{t}}\right)\right)+\left(B_{\mathrm{t}} \times p\left(c_{\mathrm{t}}\right)\right)\\
&\equiv E\left(a_{\mathrm{t}}\right)+E\left(c_{\mathrm{t}}\right)
\end{aligned} \tag{30}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned}
E\left(B_{\mathrm{t}}\right) &\equiv B_{\mathrm{t}} \times p\left(B_{\mathrm{t}}\right)\\
&\equiv (+0+1) \times p\left(B_{\mathrm{t}}\right)\\
&\equiv p\left(B_{\mathrm{t}}\right)\\
&\equiv p\left(a_{\mathrm{t}}\right)+p\left(c_{\mathrm{t}}\right)
\end{aligned} \tag{31}$$

Furthermore, it is

$$p\left(\underline{B}_{\mathrm{t}}\right) \equiv p\left(b_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right) \equiv \left(1-p\left(B_{\mathrm{t}}\right)\right) \equiv p(NotB_{\mathrm{t}}) \tag{32}$$

The expectation value E($\underline{\mathrm{B}}_{\mathrm{t}}$) is given by the equation

$$\begin{aligned}
E\left(\underline{B}_{\mathrm{t}}\right) &\equiv B_{\mathrm{t}} \times \left(1-p\left(B_{\mathrm{t}}\right)\right)\\
&\equiv B_{\mathrm{t}} \times \left(p\left(b_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right)\right)\\
&\equiv \left(B_{\mathrm{t}} \times p\left(b_{\mathrm{t}}\right)\right)+\left(B_{\mathrm{t}} \times p\left(d_{\mathrm{t}}\right)\right)\\
&\equiv E\left(b_{\mathrm{t}}\right)+E\left(d_{\mathrm{t}}\right)
\end{aligned} \tag{33}$$

Under conditions of +0/+1 distributed Bernoulli random variables it is

$$\begin{aligned} E\left(\underline{B}_{t}\right) &\equiv B_{t} \times \left(1 - p\left(B_{t}\right)\right) \\ &\equiv \left(+0+1\right) \times \left(1 - p\left(B_{t}\right)\right) \\ &\equiv \left(1 - p\left(B_{t}\right)\right) \\ &\equiv p\left(b_{t}\right) + p\left(d_{t}\right) \end{aligned} \tag{34}$$

Let $p(a_t) = p(A_t \wedge B_t)$ denote the joint probability distribution of A_t and B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E\left(a_{t}\right) &\equiv E\left(A_{t} \wedge B_{t}\right) \\ &\equiv \left(A_{t} \times B_{t}\right) \times p(A_{t} \wedge B_{t}) \\ &\equiv \left(A_{t} \times B_{t}\right) \times p(a_{t}) \end{aligned} \tag{35}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E\left(a_{t}\right) &\equiv E\left(A_{t} \wedge B_{t}\right) \\ &\equiv \left(A_{t} \times B_{t}\right) \times p(A_{t} \wedge B_{t}) \\ &\equiv \left(\left(+0+1\right) \times \left(+0+1\right)\right) \times p(A_{t} \wedge B_{t}) \\ &\equiv p(A_{t} \wedge B_{t}) \\ &\equiv p\left(a_{t}\right) \end{aligned} \tag{36}$$

Let $p(b_t) = p(A_t \wedge \neg B_t)$ denote the joint probability distribution of A_t and not B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E\left(b_{t}\right) &\equiv E\left(A_{t} \wedge \neg B_{t}\right) \\ &\equiv \left(A_{t} \times \neg B_{t}\right) \times p(A_{t} \wedge \neg B_{t}) \\ &\equiv \left(A_{t} \times \neg B_{t}\right) \times p(b_{t}) \end{aligned} \tag{37}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned} E\left(b_{t}\right) &\equiv E\left(A_{t} \wedge \neg B_{t}\right) \\ &\equiv \left(A_{t} \times \neg B_{t}\right) \times p(A_{t} \wedge \neg B_{t}) \\ &\equiv \left(\left(+0+1\right) \times \left(+0+1\right)\right) \times p(A_{t} \wedge \neg B_{t}) \\ &\equiv p(A_{t} \wedge \neg B_{t}) \\ &\equiv p\left(b_{t}\right) \end{aligned} \tag{38}$$

Let $p(c_t) = p(\neg A_t \wedge B_t)$ denote the joint probability distribution of not A_t and B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned} E\left(c_{t}\right) &\equiv E\left(\neg A_{t} \wedge B_{t}\right) \\ &\equiv \left(\neg A_{t} \wedge B_{t}\right) \times p\left(\neg A_{t} \wedge B_{t}\right) \\ &\equiv \left(\neg A_{t} \wedge B_{t}\right) \times p(c_{t}) \end{aligned} \tag{39}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned}
E(c_t) &\equiv E(\neg A_t \wedge B_t) \\
&\equiv (\neg A_t \times B_t) \times p(\neg A_t \wedge B_t) \\
&\equiv ((+0+1) \times (+0+1)) \times p(\neg A_t \wedge B_t) \\
&\equiv p(\neg A_t \wedge B_t) \\
&\equiv p(c_t)
\end{aligned} \tag{40}$$

Let $p(d_t) = p(\neg A_t \wedge \neg B_t)$ denote the joint probability distribution of not A_t and not B_t at the same Bernoulli trial (period of time) t. In general, it is

$$\begin{aligned}
E(d_t) &\equiv E(\neg A_t \times \neg B_t) \\
&\equiv (\neg A_t \times \neg B_t) \times p(\neg A_t \wedge \neg B_t) \\
&\equiv (\neg A_t \times \neg B_t) \times p(d_t)
\end{aligned} \tag{41}$$

Under conditions of +0/+1 distributed Bernoulli random variables, it is

$$\begin{aligned}
E(d_t) &\equiv E(\neg A_t \wedge \neg B_t) \\
&\equiv (\neg A_t \times \neg B_t) \times p(\neg A_t \wedge \neg B_t) \\
&\equiv ((+0+1) \times (+0+1)) \times p(\neg A_t \wedge \neg B_t) \\
&\equiv p(\neg A_t \wedge \neg B_t) \\
&\equiv p(d_t)
\end{aligned} \tag{42}$$

In general, it is

$$p(a_t) + p(b_t) + p(c_t) + p(d_t) \equiv +1 \tag{43}$$

Table 3 provide us with an overview of the definitions above.

Table 3. The two by two table of Bernoulli random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

In our understanding, it is

$$p(B_t) + p(\Lambda_t) \equiv p(a_t) + p(c_t) + p(\Lambda_t) \equiv p(a_t) + p(b_t) \equiv p(A_t) \tag{44}$$

or

$$p(c_t) + p(\Lambda_t) \equiv p(b_t) \tag{45}$$

Under conditions of Einstein's general theory of relativity, Λ denotes the Einstein cosmological (Einstein, 1917) 'constant '.

2.1.6. Binomial random variables

The binomial distribution (see Cramér, 1937) with parameters n and p has been developed by the Swiss mathematician Jakob Bernoulli (1655-1705) in a proof published in his 1713 book Ars Conjectandi (see Bernoulli, 1713) Part 1. In probability theory and statistics, the probability of getting exactly k successes in n independent Bernoulli trials is given by the probability mass function as

$$p(X_{\mathrm{t}} = k) \equiv \binom{n}{k} \cdot p^k \cdot q^{n-k} \tag{46}$$

is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ the binomial coefficient while the cumulative distribution function is given as

$$p(X_{\mathrm{t}} \leq k) \equiv 1 - p(X_{\mathrm{t}} > k) \equiv \sum_{t=0}^{k} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{47}$$

or as

$$p(X_{\mathrm{t}} > k) \equiv 1 - p(X_{\mathrm{t}} \leq k) \equiv 1 - \sum_{t=0}^{k} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{48}$$

Furthermore, it is

$$p(X_{\mathrm{t}} < k) \equiv 1 - p(X_{\mathrm{t}} \geq k) \equiv \sum_{t=0}^{k-1} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{49}$$

or

$$p(X_{\mathrm{t}} \geq k) \equiv 1 - p(X_{\mathrm{t}} < k) \equiv 1 - \sum_{t=0}^{k-1} \binom{n}{t} \cdot p^t \cdot q^{n-t} \tag{50}$$

The binomial distribution is the mathematical foundation of a binomial test. The random variable X_{t} is counting for different things. The discrete geometric (see Feller, 1950, p. 61) distribution describes under certain circumstances the number of Bernoulli trials needed to get one success. The probability that the first occurrence of success requires k independent trials, each with success probability p, is given by the equation

$$p(X_{\mathrm{t}} = k) \equiv p \cdot q^{k-1} \tag{51}$$

The negative (see Fisher, 1941, Haldane, 1941) binomial probability is a discrete probability distribution which defines the number of successes (k) in a sequence of independent and identically distributed Bernoulli trials (n) before a specified (non-random) number of failures (denoted r) occurs. The probability mass function of the negative binomial distribution is

$$p(X_{\mathrm{t}} = r) \equiv \binom{k+r-1}{k-1} p^k \cdot q^r \tag{52}$$

where k is the number of successes, r is the number of failures, and p is the probability of success.

Definition 2.10 (Expectation value and variance of a binomial random variable).

The variance(see Pearson, 1904a, p. 66) of the binomial distribution with parameters n, the number of independent experiments each asking a yes–no question and p, the probability of a single event, is defined in contrast to Pearson (see Barukčić, Ilija, 2022) as

$$\sigma\left(X_{\mathrm{t}}\right)^{2} \equiv N \times N \times p\left(X_{\mathrm{t}}\right) \times \left(1 - p\left(X_{\mathrm{t}}\right)\right) \tag{53}$$

Definition 2.11 (Two by two table of Binomial random variables).

Let a, b, c, d, A, $\underline{A}$, B, and $\underline{B}$ denote expectation values. Under conditions where *the probability of an event, an outcome, a success et cetera is **constant** from Bernoulli trial to Bernoulli trial t*, it is

$$\begin{aligned} A &= N \times E\left(A_{\mathrm{t}}\right) \\ &\equiv N \times \left(A_{\mathrm{t}} \times p\left(A_{\mathrm{t}}\right)\right) \\ &\equiv N \times \left(p\left(A_{\mathrm{t}}\right) + p\left(B_{\mathrm{t}}\right)\right) \\ &\equiv N \times p\left(A_{\mathrm{t}}\right) \end{aligned} \tag{54}$$

and

$$\begin{aligned} B &= N \times E\left(B_{\mathrm{t}}\right) \\ &\equiv N \times \left(B_{\mathrm{t}} \times p\left(B_{\mathrm{t}}\right)\right) \\ &\equiv N \times \left(p\left(A_{\mathrm{t}}\right) + p\left(c_{\mathrm{t}}\right)\right) \\ &\equiv N \times p\left(B_{\mathrm{t}}\right) \end{aligned} \tag{55}$$

where N might denote the population or even the sample size. Furthermore, it is

$$a \equiv N \times \left(E\left(A_{\mathrm{t}}\right)\right) \equiv N \times \left(p\left(A_{\mathrm{t}}\right)\right) \tag{56}$$

and

$$b \equiv N \times \left(E\left(B_{\mathrm{t}}\right)\right) \equiv N \times \left(p\left(B_{\mathrm{t}}\right)\right) \tag{57}$$

and

$$c \equiv N \times \left(E\left(c_{\mathrm{t}}\right)\right) \equiv N \times \left(p\left(c_{\mathrm{t}}\right)\right) \tag{58}$$

and

$$d \equiv N \times \left(E\left(d_{\mathrm{t}}\right)\right) \equiv N \times \left(p\left(d_{\mathrm{t}}\right)\right) \tag{59}$$

and

$$a + b + c + d \equiv A + \underline{A} \equiv B + \underline{B} \equiv N \tag{60}$$

Table 4 provide us again an overview of a two by two table of Binomial random variables.

Table 4. The two by two table of Binomial random variables

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	a	b	A
A_t	FALSE	c	d	$\underline{A}$
		B	$\underline{B}$	N

2.1.7. Independence

Definition 2.12 (Independence).

The philosophical, mathematical(Kolmogoroff, Andreĭ Nikolaevich, 1933) and physical(Einstein, 1948) concept of independence is of fundamental(Kolmogoroff, Andreĭ Nikolaevich, 1933) importance in (natural) sciences as such. In fact, it is insightful to recall again before the mind's eye Einstein's theoretical approach to the concept of independence. "*Ohne die Annahme einer ... Unabhängigkeit der ... Dinge voneinander ... wäre physikalisches Denken ... nicht möglich.*"(Einstein, 1948). In a narrower sense, the conditio sine qua non relationship concerns itself at the end only with the case whether the presence of an event A_t (condition) enables or guarantees the presence of another event B_t (conditioned). As a result of these thoughts, another question worth asking concerns the relationship between the independence of an event A_t (a condition) and another event B_t (conditioned) and the necessary condition relationship. To be confronted with the danger of bias and equally with the burden of inappropriate conclusions drawn, another fundamental question at this stage is whether is it possible that an event A_t (a condition) is a necessary condition of event B_t (conditioned) even under circumstances where the event A_t (a condition) (a necessary condition) is independent of an event B_t (conditioned)? This question is already answered more or less to the negative (Barukčić, 2018b). An event A_t which is a necessary condition of another event B_t is equally an event without which another event (B_t) could not be, could not occur, and implies as such already a kind of dependence. However, it is not mandatory that such a kind of dependence is a causal one. Thus far, **data which provide evidence of a significant conditio sine qua non relationship between two events like A_t and B_t and equally support the hypothesis that A_t and B_t are independent of each other are more or less self-contradictory and of very restricted or of none value for further analysis**. In fact, if the opposite view would be taken as plausible, contradictions are more or less inescapable. In general, an event A_t at the Bernoulli trial t need not but can be independent of the existence or of the occurrence of another event B_t *at the same* Bernoulli trial t. Mathematically(Moivre, 1718), independence (Kolmogoroff, Andreĭ Nikolaevich, 1933) in terms of probability theory is defined at the same (period of) time (i.e. Bernoulli trial) t as

$$
\begin{aligned}
p(A_t \wedge B_t) &\equiv p(A_t) \times p(B_t) \equiv p(a_t) \\
&\equiv \frac{\sum_{t=1}^{N} (A_t \wedge B_t)}{N} \equiv \frac{N \times (p(a_t))}{N} \equiv 1 - p(A_t \mid B_t) \equiv 1 - p(A_t \uparrow B_t)
\end{aligned}
\tag{61}
$$

while $p(A_{\mathrm{t}} \cap B_{\mathrm{t}})$ is the joint probability of the events A_{t} and B_{t} at a same Bernoulli trial t, $p(A_{\mathrm{t}})$ is the probability of an event A_{t} at a same Bernoulli trial t, and $p(B_{\mathrm{t}})$ is the probability of an event B_{t} at a same Bernoulli trial t. With respect to a two-by-two table (see table 3, p. 19), **under conditions of independence**, it is

$$p(b_{\mathrm{t}}) \equiv p(A_{\mathrm{t}}) \times p(\underline{B}_{\mathrm{t}}) \tag{62}$$

or

$$p(c_{\mathrm{t}}) \equiv p(\underline{A}_{\mathrm{t}}) \times p(B_{\mathrm{t}}) \tag{63}$$

and

$$p(d_{\mathrm{t}}) \equiv p(\underline{A}_{\mathrm{t}}) \times p(\underline{B}_{\mathrm{t}}) \tag{64}$$

2.1.8. Dependence

Definition 2.13 (Dependence).

The dependence of events (Barukčić, 1989, p. 57-61) is defined as

$$p\left(\underbrace{A_{\mathrm{t}} \wedge B_{\mathrm{t}} \wedge C_{\mathrm{t}} \wedge \ldots}_{n\ random\ variables}\right) \equiv \sqrt[n]{\underbrace{p(A_{\mathrm{t}}) \times p(B_{\mathrm{t}}) \times p(C_{\mathrm{t}}) \times \ldots}_{n\ random\ variables}} \tag{65}$$

2.1.9. Odds ratio (OR)

Definition 2.14 (Odds ratio (OR))**.**

Odds ratios as an appropriate measure for estimating the relative risk have become widely used in medical reports of case-control studies. The odds ratio(Fisher, 1935, p. 50) is defined(Cox, 1958) as the ratio of the odds of an event occurring in one group with respect to the odds of its occurring in another group. Odds(Yule and Pearson, 1900, p. 273) ratio (OR) is a measure of association which quantifies the relationship between two binomial distributed random variables (exposure vs. outcome) and is related to Yule's (Yule and Pearson, 1900, p. 272) Q(Yule, 1912, p. 585/586). Two events A_t and B_t are regarded as independent if $(A_t, B_t) = 1$. Let

a_t = number of persons exposed to A_t and with disease B_t

b_t = number of persons exposed to A_t but without disease $\underline{B}_t$

c_t = number of persons unexposed $\underline{A}_t$ but with disease B_t

d_t = number of persons unexposed $\underline{A}_t$: and without disease $\underline{B}_t$

a_t+c_t = total number of persons with disease B_t (case-patients)

b_t+d_t = total number of persons without disease $\underline{B}_t$ (controls).

Hereafter, consider the table 5. The odds' ratio (OR) is defined as

Table 5. The two by two table of random variables

		Conditioned/Outcome B_t		
		TRUE	FALSE	
Condition/Exposure	TRUE	a_t	b_t	A_t
A_t	FALSE	c_t	d_t	$\underline{A}_t$
		B_t	$\underline{B}_t$	N_t

$$
\begin{aligned}
OR(A_t, B_t) &\equiv \left(\frac{a_t}{b_t}\right) / \left(\frac{c_t}{d_t}\right) \\
&\equiv \left(\frac{a_t \times d_t}{b_t \times c_t}\right)
\end{aligned}
\tag{66}
$$

Remark 2.1. *Odds ratios can support logical fallacies and cause difficulties in drawing logically consistent conclusions. The chorus of voices is growing, which demand the immediate ending(Knol, 2012, Sackett, DL and Deeks, JJ and Altman, DG, 1996) of any use of Odds ratio.*

Under conditions where (b = 0), the measure of association odds ratio will collapse, because we need to divide by zero, as can be seen at eq. 66. However, according to today's rules of mathematics, *a division by zero is neither allowed nor generally accepted as possible. It does no harm to remind ourselves that in the case b = 0 the event A_t is a sufficient condition of B_t. In other words, odds ratio is not able to recognize elementary relationships of objective reality. In fact, it would be a failure not to recognize how dangerous and less valuable odds ratio is.*

Under conditions where (c = 0) odds ratio collapses too, because we need again to divide by zero, as can be seen at eq. 66. However, and again, today's rules of mathematics don't allow us a division by zero. In point of fact, in the case c = 0 it is more than necessary to point out that A_t is a necessary condition of B_t. In other words, odds ratio or the cross-product ratio is not able to recognize elementary relationships of nature like necessary conditions. We can and need to overcome all the epistemological obstacles as backed by odds ratio entirety. Sooner rather than later, we should give up this measure of relationship completely.

2.1.10. Relative risk (RR)

Relative risk ($\mathbf{RR_{nc}}$)

Definition 2.15 (Relative risk ($\mathrm{RR_{nc}}$))**.**

The degree of association between the two binomial variables can be assessed by a number of very different coefficients, the relative (Cornfield, 1951, Sadowsky et al., 1953) risk is one(Barukčić, 2021d) of them. In general, relative risk $\mathrm{RR_{nc}}$, which provides some evidence of a necessary condition, is defined as

$$\begin{aligned} RR(A_{\mathrm{t}}, B_{\mathrm{t}})_{\mathrm{nc}} &\equiv \frac{\frac{p(a_{\mathrm{t}})}{p(A_{\mathrm{t}})}}{\frac{p(c_{\mathrm{t}})}{p(NotA_{\mathrm{t}})}} \\ &\equiv \frac{p(a_{\mathrm{t}}) \times p(NotA_{\mathrm{t}})}{p(c_{\mathrm{t}}) \times p(A_{\mathrm{t}})} \\ &\equiv \frac{N \times p(a_{\mathrm{t}}) \times N \times p(NotA_{\mathrm{t}})}{N \times p(c_{\mathrm{t}}) \times N \times p(A_{\mathrm{t}})} \\ &\equiv \frac{a_{\mathrm{t}} \times (NotA_{\mathrm{t}})}{c_{\mathrm{t}} \times A_{\mathrm{t}}} \\ &\equiv \frac{EER(A_{\mathrm{t}}, B_{\mathrm{t}})}{CER(A_{\mathrm{t}}, B_{\mathrm{t}})} \end{aligned} \tag{67}$$

That what scientist generally understand by relative risk is the ratio of a probability of an event occurring with an exposure versus the probability of an event occurring without an exposure. In other words,

relative risk = (probability(event in exposed group)) / (probability(the same event in not exposed group)).

A RR(A_t,B_t) = +1 means that exposure does not affect the outcome or both are independent of each other while RR(A_t,B_t) less than +1 means that the risk of the outcome is decreased by the exposure. In this context, an RR(A_t,B_t) greater than +1 denotes that the risk of the outcome is increased by the exposure. Widely known problems with odds ratio and relative risk are already documented in literature.

Relative risk (RR (sc))

Definition 2.16 (Relative risk (RR (sc)))**.**

The relative risk (sc), which provides some evidence of a sufficient condition, is calculated from the point of view of an outcome and is defined as

$$\begin{aligned} RR(A_t,B_t)_{sc} &\equiv \frac{\frac{p(a_t)}{p(B_t)}}{\frac{p(b_t)}{p(NotB_t)}} \\ &\equiv \frac{p(a_t)\times p(NotB_t)}{p(b_t)\times p(B_t)} \\ &\equiv \frac{N\times p(a_t)\times N\times p(NotB_t)}{N\times p(b_t)\times N\times p(B_t)} \\ &\equiv \frac{a_t\times (NotB_t)}{b_t\times B_t} \\ &\equiv \frac{OPR(A_t,B_t)}{CPR(A_t,B_t)} \end{aligned} \tag{68}$$

Relative risk reduction (RRR)

Definition 2.17 (Relative risk reduction (RRR))**.**

$$\begin{aligned} RRR(A_t,B_t) &\equiv \frac{CER(A_t,B_t)-EER(A_t,B_t)}{CER(A_t,B_t)} \\ &= 1-RR(A_t,B_t) \end{aligned} \tag{69}$$

Vaccine efficacy (VE)

Definition 2.18 (Vaccine efficacy (VE))**.**

Vaccine efficacy is defined as the percentage reduction of a disease in a vaccinated group of people as compared to an unvaccinated group of people.

$$\begin{aligned} VE(A_t, B_t) &\equiv 100 \times (1 - RR(A_t, B_t)) \\ &\equiv 100 \times \left(\frac{CER(A_t, B_t) - EER(A_t, B_t)}{CER(A_t, B_t)} \right) \end{aligned} \tag{70}$$

Historically, vaccine efficacy has been designed to evaluate the efficacy of a certain vaccine by Greenwood and Yule in 1915 for the cholera and typhoid vaccines(Greenwood and Yule, 1915) and best measured using double-blind, randomized, clinical controlled trials. However, the calculated vaccine efficacy is depending too much on the study design, can lead to erroneous conclusions and is only of very limited value.

Experimental event rate (EER)

Definition 2.19 (Experimental event rate (EER))**.**

$$EER(A_t, B_t) \equiv \frac{p(a_t)}{p(A_t)} = \frac{a_t}{a_t + b_t} \tag{71}$$

Definition 2.20 (Control event rate (CER))**.**

$$CER(A_t, B_t) \equiv \frac{p(c_t)}{p(\underline{A}_t)} = \frac{c_t}{c_t + d_t} \tag{72}$$

Absolute risk reduction (ARR)

Definition 2.21 (Absolute risk reducation (ARR))**.**

$$\begin{aligned} ARR(A_t, B_t) &\equiv \frac{p(c_t)}{p(\underline{A}_t)} - \frac{p(a_t)}{p(A_t)} \\ &= \frac{c_t}{c_t + d_t} - \frac{a_t}{a_t + b_t} \\ &= CER(A_t, B_t) - EER(A_t, B_t) \end{aligned} \tag{73}$$

Absolute risk increase (ARI)

Definition 2.22 (Absolute risk increase (ARI))**.**

$$
\begin{aligned}
ARI\,(A_{\mathrm{t}}, B_{\mathrm{t}}) &\equiv \frac{p(a_{\mathrm{t}})}{p(A_{\mathrm{t}})} - \frac{p(c_{\mathrm{t}})}{p(\underline{A}_{\mathrm{t}})} \\
&= EER\,(A_{\mathrm{t}}, B_{\mathrm{t}}) - CER\,(A_{\mathrm{t}}, B_{\mathrm{t}})
\end{aligned}
\tag{74}
$$

Number needed to treat (NNT)

Definition 2.23 (Number needed to treat (NNT))**.**

$$
NNT\,(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{1}{CER\,(A_{\mathrm{t}}, B_{\mathrm{t}}) - EER\,(A_{\mathrm{t}}, B_{\mathrm{t}})} \tag{75}
$$

An ideal number needed to treat(Cook and Sackett, 1995, Laupacis et al., 1988), mathematically the reciprocal of the absolute risk reduction, is NNT = 1. Under these circumstances, everyone improves with a treatment, while no one improves with control. A higher number needed to treat indicates more or less a treatment which is less effective.

Number needed to harm (NNH)

Definition 2.24 (Number needed to harm (NNH))**.**

$$
NNH\,(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{1}{EER\,(A_{\mathrm{t}}, B_{\mathrm{t}}) - CER\,(A_{\mathrm{t}}, B_{\mathrm{t}})} \tag{76}
$$

The number needed to harm (Massel and Cruickshank, 2002), mathematically the inverse of the absolute risk increase, indicates at the end how many patients need to be exposed to a certain factor, in order to observe a harm in one patient that would not otherwise have been harmed.

Outcome prevalence rate (OPR)

Definition 2.25 (Outcome prevalence rate (OPR))**.**

$$
OPR\,(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{p(a_{\mathrm{t}})}{p(B_{\mathrm{t}})} = \frac{a_{\mathrm{t}}}{a_{\mathrm{t}} + c_{\mathrm{t}}} \tag{77}
$$

Control prevalence rate (CPR)

Definition 2.26 (Control prevalence rate (CPR))**.**

$$CPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) \equiv \frac{p(b_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} = \frac{b_{\mathrm{t}}}{b_{\mathrm{t}} + d_{\mathrm{t}}} \tag{78}$$

Bias and confounding is present to some degree in all research. In order to assess the relationship of exposure with a disease or an outcome, a fictive control group (i.e. of newborn or of young children et cetera) can be of use too. Under certain circumstances, even a CPR = 0 is imaginable.

Absolute prevalence reduction (APR)

Definition 2.27 (Absolute prevalence reduction (APR))**.**

$$APR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) \equiv CPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) - OPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) \tag{79}$$

Absolute prevalence increase (API)

Definition 2.28 (Absolute prevalence increase (API))**.**

$$API\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) \equiv OPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) - CPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) \tag{80}$$

Relative prevalence reduction (RPR)

Definition 2.29 (Relative prevalence reduction (RPR))**.**

$$\begin{aligned} RPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) &\equiv \frac{CPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right) - OPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right)}{CPR\left(A_{\mathrm{t}}, B_{\mathrm{t}}\right)} \\ &= 1 - RR(A_{\mathrm{t}}, B_{\mathrm{t}})_{\mathrm{sc}} \end{aligned} \tag{81}$$

The index NNS

Definition 2.30 (The index NNS)**.**

$$NNS(A_t, B_t) \equiv \frac{1}{CPR(A_t, B_t) - OPR(A_t, B_t)} \tag{82}$$

Mathematically, the index NNS is the reciprocal of the absolute prevalence reduction.

The index NNI

Definition 2.31 (The index NNI)**.**

$$NNI(A_t, B_t) \equiv \frac{1}{OPR(A_t, B_t) - CPR(A_t, B_t)} \tag{83}$$

Mathematically, the index NNI is the reciprocal of the absolute prevalence increase.

2.1.11. Study design and bias

Systematic observation and experimentation, inductive and deductive reasoning are essential for any formation and testing of hypotheses and theories about the natural world. In one way or another, logically and mathematically sound scientific methods and concepts are crucial constituents of any scientific progress. When all goes well, different scientists at different times and places using the same scientific methodology should be able to generate the same scientific knowledge. However, more than half (52%) of scientists surveyed believe that studies do not successfully reproduce sufficiently similar or the same results as the original studies (Baker, 2016). In a very large study on publication bias in meta-analyses, Kicinski et al. (Kicinski et al., 2015) found evidence of publication bias even in systematic reviews. Therefore, a careful re-evaluation of the study/experimental design, the statistical methods and other scientific means which underpin scientific inquiry and research goals appears to be necessary once and again. While it is important to recognize the shortcoming of today's science, one issue which has shaped debates over studies published is the question: **has a study really measured what it set out to**? Even if studies carried out can vary greatly in detail, the data from the studies itself provide information about the credibility of the data.

Index of unfairness (IOU)

Definition 2.32 (Index of unfairness).

The index of unfairness (Barukčić, 2019c) (IOU) is defined as

$$p\left(IOU\left(A,B\right)\right) \equiv Absolute\left(\left(\frac{A+B}{N}\right)-1\right) \tag{84}$$

A very good study design should assure as much as possible a p(IOU) = 0. In point of fact, against the background of lacking enough experience with the use of p(IOU), a p(IOU) up to 0.25 could be of use too. An index of unfairness is of use to prove whether sample data are biased and whether sample data can be used for Chi-square based analysis of necessary conditions, of sufficient conditions and of causal relationships.

Index of independence (IOI)

Definition 2.33 (Index of independence).

The index of independence(Barukčić, 2019b) (IOI) is defined as

$$p\left(IOI\left(A_{t},\underline{B}_{t}\right)\right) \equiv Absolute\left(\left(\frac{A_{t}+\underline{B}_{t}}{N}\right)-1\right) \tag{85}$$

or as

$$p\left(IOI\left(\underline{A}_{t},B_{t}\right)\right) \equiv Absolute\left(\left(\frac{\underline{A}_{t}+B_{t}}{N}\right)-1\right) \tag{86}$$

A very good study design which aims to prove **an exclusion relationship or a causal relationship** should assure as much as possible a p(IOI) = 0. However, once again, against the background of lacking enough experience with the use of p(IOI), sample data with a p(IOI) up to 0.25 are of use too. Today, most double-blind placebo-controlled studies are based on the demand that **p(IOU) = p(IOI)** while the value of p(IOU) of has been widely neglected. Such an approach leads to unnecessary big sample sizes, the increase of cost, the waste of time and, most importantly of all, to epistemological systematically biased sample data and conclusions drawn. A change is necessary.

Index of relationship (IOR)

Definition 2.34 (Index of relationship (IOR))**.**

Due to several reasons, it is not always easy to identify the unique characteristics between two events like A_t and B_t. And more than that, it is difficult to decide what to do, and much more difficult to know in which direction one should think and which decision is right. Sometimes it is helpful to know at least something about the direction of the relationship between two events like A_t and B_t. Under conditions where $p(a_{t}) = p(A_{t} \wedge B_{t})$, the index of relationship(Barukčić, 2021b), abbreviated as IOR, is defined as

$$
\begin{aligned}
IOR(A_{\mathrm{t}}, B_{\mathrm{t}}) &\equiv \left(\frac{p(A_{\mathrm{t}} \wedge B_{\mathrm{t}})}{p(B_{\mathrm{t}}) \times p(A_{\mathrm{t}})}\right) - 1 \\
&\equiv \left(\frac{p(a_{\mathrm{t}})}{p(B_{\mathrm{t}}) \times p(A_{\mathrm{t}})}\right) - 1 \\
&\equiv \left(\left(\frac{N \times N \times p(a_{\mathrm{t}})}{N \times p(B_{\mathrm{t}}) \times N \times p(A_{\mathrm{t}})}\right) - 1\right) \\
&\equiv \left(\left(\frac{N \times a}{A \times B}\right) - 1\right)
\end{aligned}
\tag{87}
$$

where $p(A_t)$ denotes the probability of an event A_t at the Bernoulli trial t and $p(B_t)$ denotes the probability of another event B_t at the same Bernoulli trial t while $p(a_t)$ denotes the joint probability of $p(A_t$ AND $B_t)$ at the same Bernoulli trial t and a, A and B may denote the expectation values.

2.2.1. Exclusion relationship

Definition 2.35 (Exclusion relationship [*EXCL*]).

Mathematically, the exclusion (EXCL) relationship, denoted by $p(A_t \mid B_t)$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$
\begin{aligned}
p(A_t \mid B_t) &\equiv p(A_t \uparrow B_t) \\
&\equiv p(b_t) + p(c_t) + p(d_t) \\
&\equiv \frac{N \times (p(b_t) + p(c_t) + p(d_t))}{N} \\
&\equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \vee \underline{B}_t)}{N} \equiv \frac{b+c+d}{N} \\
&\equiv \frac{b+\underline{A}}{N} \\
&\equiv \frac{c+\underline{B}}{N} \\
&\equiv +1
\end{aligned}
\tag{88}
$$

Based on the 1913 Henry Maurice Sheffer (1882-1964) relationship, the Sheffer stroke(Nicod, 1917, Sheffer, 1913) usually denoted by $\uparrow$, it is $p(A_t \wedge B_t) \equiv 1 - p(A_t \mid B_t)$ (see table 6).

Table 6. A_t excludes B_t and vice versa.

		Conditioned (COVID-19) B_t		
		TRUE	FALSE	
Condition (Vaccine)	TRUE	**+0**	$p(b_t)$	$p(A_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

Example 2.1. *Pfizer Inc. and BioNTech SE announced on Monday, November 09, 2020 - 06:45am results from a Phase 3 COVID-19 vaccine trial with 43.538 participants which provides evidence that their vaccine (BNT162b2) is preventing COVID-19 in participants without evidence of prior SARS-CoV-2 infection. In toto, 170 confirmed cases of COVID-19 were evaluated, with 8 in the vaccine group versus 162 in the placebo group. The exclusion relationship can be calculated as follows.*

$$
\begin{aligned}
p(Vaccine : BNT162b2 \mid COVID-19(infection)) &\equiv p(b_t) + p(c_t) + p(d_t) \\
&\equiv 1 - p(a_t) \\
&\equiv 1 - \left(\frac{8}{43538}\right) \\
&\equiv +0,99981625
\end{aligned}
\tag{89}
$$

with a P Value = 0,000184.

Following Kolmogorov's definition of an n-dimensional probability density (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) of random variables A_t, B_t et cetera at the point t, we obtain

$$\begin{aligned} p(A_t \mid B_t) &\equiv p(\underline{U}_t \cup \underline{W}_t) \\ &\equiv 1 - p(A_t \cap B_t) \\ &\equiv 1 - \int_{-\infty}^{A_t} \int_{-\infty}^{B_t} f(A_t, B_t)\ dA_t\, dB_t \\ &\equiv +1 \end{aligned} \tag{90}$$

while $p(A_t \mid B_t)$ would denote the cumulative distribution function of random variables and $f(A_t, B_t)$ is the joint density function.

2.2.2. Observational study and exclusion relationship

Under conditions of an observational study, the exclusion relationship follows approximately(see Barukčić, 2021a) as

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \geq 1 - \frac{p(a_t)}{p(B_t)} \tag{91}$$

2.2.3. Experimental study and exclusion relationship

Under conditions of an experimental study, the exclusion relationship follows approximately(see Barukčić, 2021a) as

$$p(A_t \mid B_t) \equiv p(A_t \uparrow B_t) \geq 1 - \frac{p(a_t)}{p(A_t)} \tag{92}$$

2.2.4. The goodness of fit test of an exclusion relationship

Definition 2.36 (The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship).

Under some well known circumstances, testing hypothesis about an exclusion relationship p(A_t | B_t) is possible by the chi-square(see Pearson, 1900b) distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an exclusion relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(A_{\text{t}} \mid B_{\text{t}}\right) \mid A\right) &\equiv \frac{(b-(a+b))^2}{A} + \\
&\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\
&\equiv \frac{a^2}{A} + 0 \\
&\equiv \frac{a^2}{A}
\end{aligned} \tag{93}
$$

or equally as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(A_{\text{t}} \mid B_{\text{t}}\right) \mid B\right) &\equiv \frac{(c-(a+c))^2}{B} + \\
&\quad \frac{((b+d)-\underline{B})^2}{\underline{B}} \\
&\equiv \frac{a^2}{B} + 0 \\
&\equiv \frac{a^2}{B}
\end{aligned} \tag{94}
$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of an exclusion relationship/distribution $p(A_t \mid B_t)$, in which case the null hypothesis has to be accepted. Yate's (Yates, 1934) continuity correction was not used under these circumstances.

2.2.5. The left-tailed p Value of an exclusion relationship

Definition 2.37 (The left-tailed p Value of an exclusion relationship).

It is known that as a sample size, N, increases, a sampling distribution of a special test statistic approaches the normal distribution (central limit theorem). Under these circumstances, the left-tailed (lt) p Value (Barukčić, 2019d) of an exclusion relationship can be calculated as follows.

$$
\begin{aligned}
pValue_{\text{lt}}\left(A_{\text{t}} \mid B_{\text{t}}\right) &\equiv 1 - e^{-\left(1-p\left(A_{\text{t}} \mid B_{\text{t}}\right)\right)} \\
&\equiv 1 - e^{-(a/N)}
\end{aligned} \tag{95}
$$

A low p-value may provide some evidence of statistical significance.

2.2.6. Neither nor conditions

Definition 2.38 (Neither A_t nor B_t conditions [*NOR*]).

Mathematically, a neither A_t nor B_t condition (or rejection according to the French philosopher and logician Jean George Pierre Nicod (1893-1924), i.e. Jean Nicod's statement (Nicod, 1924)) relationship (NOR), denoted by $p(A_t \downarrow B_t)$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p(A_t \downarrow B_t) &\equiv p(d_t) \\ &\equiv \frac{N - \sum_{t=1}^{N} (A_t \vee B_t)}{N} \equiv \frac{\sum_{t=1}^{N} (\underline{A}_t \wedge \underline{B}_t)}{N} \equiv \frac{N \times (p(d_t))}{N} \\ &\equiv \frac{d}{N} \\ &\equiv +1 \end{aligned} \tag{96}$$

2.2.7. The Chi square goodness of fit test of a neither nor condition relationship

Definition 2.39 (**The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship**).

A neither A_t nor B_t condition relationship $p(A_t \downarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution). The $\tilde{\chi}^2$ goodness of fit test of a neither A_t nor B_t condition relationship with degree of freedom (d. f.) of d. f. = 1 may be calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}((A_t \downarrow B_t) \mid A) &\equiv \frac{(d-(c+d))^2}{\underline{A}} + \\ &\quad \frac{((a+b)-A)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}} + 0 \end{aligned} \tag{97}$$

or equally as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}((A_t \downarrow B_t) \mid B) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\ &\quad \frac{((a+c)-B)^2}{B} \\ &\equiv \frac{b^2}{\underline{B}} + 0 \end{aligned} \tag{98}$$

Yate's (Yates, 1934) continuity correction has not been used in this context.

2.2.8. The left-tailed p Value of a neither nor B condition relationship

Definition 2.40 (**The left-tailed p Value of a neither A_t nor B_t condition relationship**).

The left-tailed (lt) p Value (Barukčić, 2019d) of a neither A_t nor B_t condition relationship can be calculated as follows.

$$
\begin{aligned}
pValue_{lt}(A_t \downarrow B_t) &\equiv 1 - e^{-(1-p(A_t \downarrow B_t))} \\
&\equiv 1 - e^{-p(A_t \vee B_t)} \\
&\equiv 1 - e^{-((a+b+c)/N)}
\end{aligned}
\tag{99}
$$

where $\vee$ may denote disjunction or logical inclusive or. In this context, a low p-value indicates again a statistical significance. In general, it is $p(A_t \vee B_t) \equiv 1 - p(A_t \downarrow B_t)$ (see table 7).

Table 7. Neither A_t nor B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	0	0
	NO	0	1	1
		0	1	1

2.2.9. Necessary condition

Definition 2.41 (**Necessary condition [*Conditio sine qua non*]**).

Despite the most extended efforts, the current state of research on conditions and conditioned is still incomplete and very contradictory. However, even thousands of years ago and independently of any human mind and consciousness, water has been and is still a necessary condition for (human) life. Without water, there has been and there is no (human) life. It comes therefore as no surprise that one of the first documented attempts to present a rigorous theory of conditions and causation (see also Aristotle et al., 1908, Metaphysica III 2 997a 10 and 13/14) came from the Greek philosopher and scientist Aristotle (384-322 BCE). Thus far, it is amazing that Aristotle himself made already a strict distinction between conditions and causes. Taking Aristotle very seriously, it is necessary to consider that

> "... everything which has a ... ... potency in question ... ... has the potency ... of acting ...
>
> not in all circumstances but on certain conditions ... "
>
> (see also Aristotle et al., 1908, Metaphysica IX 5 1048a 14-19)

Before going into details, Aristotle went on to define the necessary condition as follows.

"... necessary ... means ...

without ... a condition, a thing cannot live ... "

(see also Aristotle et al., 1908, Metaphysica V 2 1015a 20-22)

In point of fact, Aristotle developed a theory of conditions and causality commonly referred to as the doctrine of four causes. Many aspects and general features of Aristotle's logical concept of causality are meanwhile extensively and critically debated in secondary literature. However, even if the Greek philosophers Heraclitus, Plato, Aristotle et cetera numbers among the greatest philosophers of all time, the philosophy has evolved. Scientific knowledge and objective reality are deeply interrelated and cannot be reduced only to Greek philosophers like Aristotle. As mentioned at the start of the article, the specification of necessary conditions has traditionally been part of the philosopher's investigations of different phenomena. Behind the need of a detailed evidence, it is justified to consider that philosophy or philosophers as such certainly do not possess **a monopoly on the truth** and other areas such as medicine as well as other sciences and technology may transmit truths as well and may be of help to move beyond one's self enclosed unit. Seemingly, **the law's concept of causation** justifies to say few words on this subject, to put some light on some questions. Are there any criteria in law for deciding whether one action or an event A_t has caused another (generally harmful) event B_t? What are these criteria? May causation in legal contexts differ from causation outside the law, for example, in science or in our everyday life and to what extent? Under which circumstances is it justified to tolerate such differences as may be found to exist? To understand just what is the law's concept of causation, it is useful to know how the highest court of states is dealing with causation. In the case *Hayes v. Michigan Central R. Co., 111 U.S. 228*, the U.S. Supreme Court defined 1884 conditio sine qua non as follows: "... **causa sine qua non – a cause which, if it had not existed, the injury would not have taken place**". (Justice Matthews, Mr., 1884) The German Bundesgerichtshof für Strafsachen stressed once again the importance of conditio sine qua non relationship in his decision by defining the following: "**Ursache eines strafrechtlich bedeutsamen Erfolges jede Bedingung, die nicht hinweggedacht werden kann, ohne daß der Erfolg entfiele**"(Bundesgerichtshof für Strafsachen, 1951) Another lawyer elaborated on the basic issue of **identity and difference between cause and condition**. Von Bar was writing: "Die erste Voraussetzung, welche erforderlich ist, damit eine Erscheinung als die Ursache einer anderen bezeichnet werden könne, ist, daß jene eine der Bedingungen dieser sein. Würde die zweite Erscheinung auch dann eingetreten sein, wenn die erste nicht vorhanden war, so ist sie in keinem Falle Bedingung und noch weniger Ursache. Wo immer ein Kausalzusammenhang behauptet wird, da muß er wenigstens diese Probe aushalten ... **Jede Ursache ist nothwendig auch eine Bedingung eines Ereignisses; aber nicht jede Bedingung ist Ursache zu nennen**."(Bar, 1871) Von Bar's position translated into English: *The first requirement, which is required, thus that something could be called as the cause of another, is that the one has to be one of the conditions of the other. If the second something had occurred even if the first one did not exist, so it is by no means a condition and still less a cause. Wherever a causal relationship is claimed, the same must at least withstand this test... Every cause is necessarily also a condition of an event too; but not every condition is cause* *too*. Thus far, let us consider among other the following in order to specify necessary conditions from another, probabilistic point of view. An event (i.e. A_t) which is a necessary condition of another event or outcome (i.e. B_t) must be given, must be present for a conditioned, for an event or for an outcome B_t to occur. A necessary condition (i.e. A_t) is a requirement which must be fulfilled **at every single Bernoulli trial t**, in order for a conditioned or an outcome (i.e. B_t) to occur, but it alone does not determine the occurrence of an event. In other words, if a necessary condition (i.e. A_t) is given, an outcome (i.e. B_t) need not occur. In contrast to a necessary condition, a 'sufficient'condition is the one condition which 'guarantees'that an outcome will take place or must occur for sure. Under which conditions we may infer about the unobserved and whether observations made are able at all to justify predictions about potential observations which have not yet been made or even general claims which my go even beyond the observed (*the 'problem of induction')* is not the issue of the discussion at this point. Besides of the principal necessity of meeting such a challenge, a necessary condition of an event can but need not be at the same Bernoulli trial t a sufficient condition for an event to occur. However, theoretically, it is possible that an event or an outcome is determined by many necessary conditions. Let us focus to some extent on what this means, or in other words how much importance can we attribute to such a special case. *Example*. A human being cannot live without oxygen. A human being cannot live without water. A human being cannot live without a brain. A human being cannot live without kidneys. A human being cannot live without ... et cetera. Thus far, even if oxygen is given, if water is given, if a brain is given, without functioning kidney's (or something similar) a human being will not survive on the long run. This example is of use to reach the following conclusion. Although it might seem somewhat paradoxical at first sight, **even under circumstances where a condition or an outcome depends on several different necessary conditions it is particularly important that every single of these necessary conditions for itself must be given otherwise the conditioned (i.e. the outcome) will not occur**. Mathematically, the necessary condition (SINE) relationship, denoted by $p(A_t \leftarrow B_t)$ in terms of statistics and probability theory, is defined (Barukčić, 1989, p. 15-28) as

$$
\begin{aligned}
p\left(A_t \leftarrow B_t\right) \equiv p\left(A_t \vee \underline{B}_t\right) &\equiv \frac{\sum_{t=1}^{N}\left(A_t \vee \underline{B}_t\right)}{N} \equiv \frac{\left(A_t \vee \underline{B}_t\right) \times p\left(A_t \vee \underline{B}_t\right)}{\left(A_t \vee \underline{B}_t\right)} \\
&\equiv p\left(a_t\right)+p\left(b_t\right)+p\left(d_t\right) \\
&\equiv \frac{N \times\left(p\left(a_t\right)+p\left(b_t\right)+p\left(d_t\right)\right)}{N} \equiv \frac{E\left(A_t \leftarrow B_t\right)}{N} \\
&\equiv \frac{a+b+d}{N} \equiv \frac{E\left(A_t \vee \underline{B}_t\right)}{N} \\
&\equiv \frac{A+d}{N} \equiv \frac{E\left(A_t \leftarrow B_t\right)}{N} \\
&\equiv \frac{a+\underline{B}}{N} \equiv \frac{E\left(A_t \vee \underline{B}_t\right)}{N} \\
&\equiv +1
\end{aligned} \tag{100}
$$

where $E\left(A_t \leftarrow B_t\right) \equiv E\left(A_t \vee \underline{B}_t\right)$ indicates the expectation value of the necessary condition. In general, it is $p\left(A_t \prec B_t\right) \equiv 1 - p\left(A_t \leftarrow B_t\right)$ (see Table 8).

Remark 2.2. *A necessary condition A_t is characterized itself by the property that another event B_t will not occur if A_t is not given, if A_t did not occur (Barukčić, 1989, 1997, 2005, 2016b, 2017b,c,*

Table 8. Necessary condition.

		Conditioned B_t		
		TRUE	FALSE	
Condition	TRUE	$p(a_t)$	$p(b_t)$	$p(A_t)$
A_t	FALSE	**+0**	$p(d_t)$	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

*2020a,b,c,d, Barukčić and Ufuoma, 2020). **Example**. Once again, a human being cannot live without water. A human being cannot live without gaseous oxygen, et cetera. Water itself is a necessary condition for human life. However, gaseous oxygen is a necessary condition for human life too. Thus far, even if water is given and even if water is a necessary condition for human life, without gaseous oxygen there will be no human life. In general, if a conditioned or an outcome B_t depends on the necessary condition A_t and equally on numerous other necessary conditions, an event B_t will not occur if A_t itself is not given independently of the occurrence of other necessary conditions.*

Taking into account Kolmogorov's definition of an n-dimensional probability density (see also Kolmogorov, Andreĭ Nikolaevich, 1950, p. 26) of random variables A_t, B_t et cetera at the (period of) time t, we obtain

$$
\begin{aligned}
p\left(A_t \leftarrow B_t\right) &\equiv +1 \\
&\equiv +1 - p\left(c_t\right) \\
&\equiv +1 - p\left(\underline{A}_t \cap B_t\right) \\
&\equiv \left(\int_{-\infty}^{A_t}\int_{-\infty}^{B_t} f\left(A_t, B_t\right)\, dA_t\, dB_t\right) + \left(1 - \int_{-\infty}^{B_t} f\left(B_t\right)\, dB_t\right)
\end{aligned}
\tag{101}
$$

while $p\left(A_t \leftarrow B_t\right)$ would denote the cumulative distribution function of random variables of a necessary condition. Another adequate formulation of a necessary condition is possible too. If certain conditions are met, then necessary conditions and sufficient conditions are one way or another converses (see Gomes, 2009) of each other, too. It is

$$
p\left(A_t \leftarrow B_t\right) \equiv \underbrace{\left(A_t \vee \underline{B}_t\right)}_{\text{(Nessessary condition)}} \equiv \underbrace{\left(\underline{B}_t \vee A_t\right)}_{\text{(Sufficient condition)}} \equiv p\left(B_t \rightarrow A_t\right) \tag{102}
$$

There are circumstances under which

$$
p\left(A_t \leftarrow B_t\right) \equiv \underbrace{\left(A_t \vee \underline{B}_t\right)}_{\text{(Necessary condition)}} \equiv \underbrace{\left(\underline{A}_t \vee B_t\right)}_{\text{(Sufficient condition)}} \equiv p\left(A_t \rightarrow B_t\right) \tag{103}
$$

However, equation 102 does not imply the relationship of equation 103 under any circumstances.

2.2.10. The Chi-square goodness of fit test of a necessary condition relationship

Definition 2.42 (The $\tilde{\chi}^2$ goodness of fit test of a necessary condition relationship).

Under some well known circumstances, hypothesis about the conditio sine qua non relationship $p(A_t \leftarrow B_t)$ can be tested by the chi-square distribution (also chi-squared or χ^2-distribution), first described by the German statistician Friedrich Robert Helmert (Helmert, 1876) and later rediscovered by Karl Pearson (Pearson, 1900a) in the context of a goodness of fit test. The $\tilde{\chi}^2$ goodness of fit test of a conditio sine qua non relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}(A_t \leftarrow B_t \mid B) &\equiv \frac{(a-(a+c))^2}{B} + \\ &\quad \frac{((b+d)-\underline{B})^2}{\underline{B}} \\ &\equiv \frac{c^2}{B} + 0 \\ &\equiv \frac{c^2}{B}\end{aligned} \tag{104}$$

or equally as

$$\begin{aligned}\tilde{\chi}^2{}_{\text{Calculated}}(A_t \leftarrow B_t \mid \underline{A}) &\equiv \frac{(d-(c+d))^2}{\underline{A}} + \\ &\quad \frac{((a+b)-A)^2}{A} \\ &\equiv \frac{c^2}{\underline{A}} + 0 \\ &\equiv \frac{c^2}{\underline{A}}\end{aligned} \tag{105}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. It has not yet been finally clarified whether the use of Yate's (Yates, 1934) continuity correction is necessary at all.

2.2.11. The left-tailed p Value of the conditio sine qua non relationship

Definition 2.43 (The left-tailed p Value of the conditio sine qua non relationship).

The left-tailed (lt) p Value (Barukčić, 2019d) of the conditio sine qua non relationship can be calculated as follows.

$$pValue_{\mathrm{lt}}\left(A_{\mathrm{t}} \leftarrow B_{\mathrm{t}}\right) \equiv 1-e^{-\left(1-p\left(A_{\mathrm{t}} \leftarrow B_{\mathrm{t}}\right)\right)} \\ \equiv 1-e^{-(c/N)} \tag{106}$$

2.2.12. Sufficient condition

Definition 2.44 (Sufficient condition [*Conditio per quam*]).

Mathematically, the sufficient condition (IMP) relationship, denoted by $\mathrm{p}(\mathrm{A_t} \rightarrow \mathrm{B_t})$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right) \equiv p\left(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}\right) \equiv \frac{\sum_{t=1}^{N}\left(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}\right)}{N} \equiv \frac{\left(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}\right) \times p\left(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}\right)}{\left(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}\right)} \\ \equiv p\left(a_{\mathrm{t}}\right)+p\left(c_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right) \\ \frac{N \times\left(p\left(a_{\mathrm{t}}\right)+p\left(c_{\mathrm{t}}\right)+p\left(d_{\mathrm{t}}\right)\right)}{N} \\ \equiv \frac{a+c+d}{N} \equiv \frac{E\left(\underline{A}_{\mathrm{t}} \vee B_{\mathrm{t}}\right)}{N} \\ \equiv \frac{B+d}{N} \equiv \frac{E\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right)}{N} \\ \equiv \frac{a+\underline{A}}{N} \\ \equiv +1 \end{aligned} \tag{107}$$

It is $p\left(A_{\mathrm{t}} \succ\!\!- B_{\mathrm{t}}\right) \equiv 1-p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right)$ (see Table 9).

Table 9. Sufficient condition.

		Conditioned $\mathrm{B_t}$		
		TRUE	FALSE	
Condition	TRUE	$\mathrm{p(a_t)}$	**+0**	$\mathrm{p(A_t)}$
$\mathrm{A_t}$	FALSE	$\mathrm{p(c_t)}$	$\mathrm{p(d_t)}$	$\mathrm{p(\underline{A}_t)}$
		$\mathrm{p(B_t)}$	$\mathrm{p(\underline{B}_t)}$	+1

Remark 2.3. *A sufficient condition A_t is characterized by the property that another event B_t will occur if A_t is given, if A_t itself occured (Barukčić, 1989, 1997, 2005, 2016b, 2017b,c, 2020a,b,c,d, Barukčić and Ufuoma, 2020). **Example**. The ground, the streets, the trees, human beings and many other objects too will become wet during heavy rain. Especially, **if** it is raining (event A_t), **then** human beings will become wet (event B_t). However, even if this is a common human wisdom, a human being equipped with*

an appropriate umbrella (denoted by R_t) need not become wet even during heavy rain. An appropriate umbrella (R_t) is similar to an event with the potential to counteract the occurrence of another event (B_t) and can be understood something as an ***anti-dot*** *of another event. In other words, an appropriate umbrella is an antidote of the effect of rain on human body, an appropriate umbrella has the potential to protect humans from the effect of rain on their body. It is a good rule of thumb that the following relationship*

$$p(A_t \rightarrow B_t) + p(R_t \wedge B_t) \equiv +1 \tag{108}$$

indicates that R_t is an antidote of A_t. However, taking a shower, swimming in a lake et cetera may make human hair wet too. More than anything else, however, these events does not affect the final outcome, the effect of raining on human body.

2.2.13. The Chi square goodness of fit test of a sufficient condition relationship

Definition 2.45 (**The $\tilde{\chi}^2$ goodness of fit test of a sufficient condition relationship**).

Under some well known circumstances, testing hypothesis about the conditio per quam relationship $\mathrm{p}(\mathrm{A_t} \rightarrow \mathrm{B_t})$ is possible by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a conditio per quam relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(A_{\text{t}} \rightarrow B_{\text{t}} \mid A) &\equiv \frac{(a-(a+b))^2}{A} + \\ &\quad \frac{((c+d)-\underline{A})^2}{\underline{A}} \\ &\equiv \frac{b^2}{A} + 0 \\ &\equiv \frac{b^2}{A} \end{aligned} \tag{109}$$

or equally as

$$\begin{aligned} \tilde{\chi}^2{}_{\text{Calculated}}(A_{\text{t}} \rightarrow B_{\text{t}} \mid \underline{B}) &\equiv \frac{(d-(b+d))^2}{\underline{B}} + \\ &\quad \frac{((a+c)-B)^2}{B} \\ &\equiv \frac{b^2}{\underline{B}} + 0 \\ &\equiv \frac{b^2}{\underline{B}} \end{aligned} \tag{110}$$

and can be compared with a theoretical chi-square value at a certain level of significance α. The $\tilde{\chi}^2$-distribution equals zero when the observed values are equal to the expected/theoretical values of the conditio per quam relationship/distribution p($A_t \rightarrow B_t$), in which case the null hypothesis is accepted. Yate's (Yates, 1934) continuity correction has not been used in this context.

2.2.14. The left-tailed p Value of the conditio per quam relationship

Definition 2.46 (The left-tailed p Value of the conditio per quam relationship).

The left-tailed (lt) p Value (Barukčić, 2019d) of the conditio per quam relationship can be calculated as follows.

$$\begin{aligned} pValue_{lt}(A_t \rightarrow B_t) &\equiv 1 - e^{-(1-p(A_t \rightarrow B_t))} \\ &\equiv 1 - e^{-(b/N)} \end{aligned} \tag{111}$$

Again, a low p-value indicates a statistical significance.

2.2.15. Necessary and sufficient conditions

Definition 2.47 (Necessary and sufficient conditions [*EQV*]).

The necessary and sufficient condition (EQV) relationship, denoted by p($A_t \leftrightarrow B_t$) in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$\begin{aligned} p(A_t \leftrightarrow B_t) &\equiv \frac{\sum_{t=1}^{N} ((A_t \vee \underline{B}_t) \wedge (\underline{A}_t \vee B_t))}{N} \\ &\equiv p(a_t) + p(d_t) \\ &\equiv \frac{N \times (p(a_t) + p(d_t))}{N} \\ &\equiv \frac{a+d}{N} \\ &\equiv +1 \end{aligned} \tag{112}$$

2.2.16. The Chi square goodness of fit test of a necessary and sufficient condition relationship

Definition 2.48 (The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship).

Even the necessary and sufficient condition relationship p($A_t \leftrightarrow B_t$) can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\text{t}} \leftrightarrow B_{\text{t}} \mid A\right) \equiv \frac{(a-(a+b))^2}{A} + \frac{d-((c+d))^2}{\underline{A}} \equiv \frac{b^2}{A} + \frac{c^2}{\underline{A}} \tag{113}$$

or equally as

$$\tilde{\chi}^2{}_{\text{Calculated}}\left(A_{\text{t}} \leftrightarrow B_{\text{t}} \mid B\right) \equiv \frac{(a-(a+c))^2}{B} + \frac{d-((b+d))^2}{\underline{B}} \equiv \frac{c^2}{B} + \frac{b^2}{\underline{B}} \tag{114}$$

The calculated $\tilde{\chi}^2$ goodness of fit test of a necessary and sufficient condition relationship can be compared with a theoretical chi-square value at a certain level of significance α. Under conditions where the observed values are equal to the expected/theoretical values of a necessary and sufficient condition relationship/distribution $p(A_t \leftrightarrow B_t)$, the $\tilde{\chi}^2$-distribution equals zero. It is to be cleared whether Yate's (Yates, 1934) continuity correction should be used at all.

2.2.17. The left-tailed p Value of a necessary and sufficient condition relationship

Definition 2.49 (The left-tailed p Value of a necessary and sufficient condition relationship).

The left-tailed (lt) p Value (Barukčić, 2019d) of a necessary and sufficient condition relationship can be calculated as follows.

$$pValue_{\text{lt}}\left(A_{\text{t}} \leftrightarrow B_{\text{t}}\right) \equiv 1 - e^{-(1-p(A_{\text{t}} \leftrightarrow B_{\text{t}}))} \equiv 1 - e^{-((b+c)/N)} \tag{115}$$

In this context, a low p-value indicates again a statistical significance. Table 10 may provide an overview of the theoretical distribution of a necessary and sufficient condition.

Table 10. Necessary and sufficient condition.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	1	0	1
	NO	0	1	1
		1	1	2

2.2.18. Either or conditions

Definition 2.50 (Either A_t or B_t conditions [*NEQV*]).

Mathematically, an either A_t or B_t condition relationship (NEQV), denoted by $p(A_t >\!\!-\!\!< B_t)$ in terms of statistics and probability theory, is defined(Barukčić, 1989, p. 68-70) as

$$
\begin{aligned}
p\left(A_t >\!\!-\!\!< B_t\right) &\equiv \frac{\sum\limits_{t=1}^{N}\left(\left(A_t \wedge \underline{B}_t\right) \vee \left(\underline{A}_t \wedge B_t\right)\right)}{N} \\
&\equiv p\left(b_t\right) + p\left(c_t\right) \\
&\equiv \frac{N \times \left(p\left(b_t\right) + p\left(c_t\right)\right)}{N} \\
&\equiv \frac{b+c}{N} \\
&\equiv +1
\end{aligned}
\tag{116}
$$

It is $p\left(A_t >\!\!-\!\!< B_t\right) \equiv 1 - p\left(A_t <\!\!-\!\!> B_t\right)$ (see Table 11).

Table 11. Either A_t or B_t relationship.

		Conditioned B_t		
		YES	NO	
Condition A_t	YES	0	1	1
	NO	1	0	1
		1	1	2

2.2.19. The Chi-square goodness of fit test of an either or condition relationship

Definition 2.51 (The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship).

An either or condition relationship $p(A_t >\!\!-\!\!< B_t)$ can be tested by the chi-square distribution (also chi-squared or $\tilde{\chi}^2$-distribution) too. The $\tilde{\chi}^2$ goodness of fit test of an either or condition relationship with degree of freedom (d. f.) of d. f. = 1 is calculated as

$$
\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(A_t >\!\!-\!\!< B_t\right) \mid A\right) &\equiv \frac{\left(b-\left(a+b\right)\right)^2}{A} + \\
&\quad \frac{c-\left(\left(c+d\right)\right)^2}{\underline{A}} \\
&\equiv \frac{a^2}{A} + \frac{d^2}{\underline{A}}
\end{aligned}
\tag{117}
$$

or equally as

$$\begin{aligned}
\tilde{\chi}^2{}_{\text{Calculated}}\left(\left(A_{\text{t}} \succ\!\prec B_{\text{t}} \right) \mid B \right) &\equiv \frac{\left(c - \left(a + c \right) \right)^2}{B} + \\
&\quad \frac{b - \left(\left(b + d \right) \right)^2}{\underline{B}} \\
&\equiv \frac{a^2}{B} + \frac{d^2}{\underline{B}}
\end{aligned} \tag{118}$$

Yate's (Yates, 1934) continuity correction has not been used in this context.

2.2.20. The left-tailed p Value of an either or condition relationship

Definition 2.52 (The left-tailed p Value of an either or condition relationship).

The left-tailed (lt) p Value (Barukčić, 2019d) of an either or condition relationship can be calculated as follows.

$$\begin{aligned}
pValue_{\text{lt}}\left(A_{\text{t}} \succ\!\prec B_{\text{t}} \right) &\equiv 1 - e^{-\left(1 - p\left(A_{\text{t}} > - < B_{\text{t}}\right)\right)} \\
&\equiv 1 - e^{-\left(\left(a + d\right)/N\right)}
\end{aligned} \tag{119}$$

In this context, a low p-value indicates again a statistical significance.

2.2.21. Causal relationship k

The history of the denialism of causality in Philosophy, Mathematics, Statistics, Physics et cetera is very long. We only recall David Hume's (1711-1776) account of causation and his inappropriate reduction of the cause-effect relationship to a simple habitual connection in human thinking or Immanuel Kant's (1724-1804) initiated trial to consider causality as nothing more but a '*a priori*'given category (Langsam, 1994) in human reasoning and other similar attempts too. It is worth noting in this context that especially Karl Pearson (1857 - 1936) himself has been engaged in a long lasting and never-ending crusade against causation too. "**Pearson categorically denies the need for an independent concept of causal relation beyond correlation ... he *exterminated* causation from statistics before it had a chance to take root** "(Pearl, 2000) At the beginning of the 20th century notable proponents of **conditionalism** like the German anatomist and pathologist David Paul von Hansemann (Hansemann, David Paul von, 1912) (1858 - 1920) and the biologist and physiologist Max Richard Constantin Verworn(Verworn, 1912) (1863 - 1921) started a new attack(Kröber, 1961) on the principle of causality. In his essay "Kausale und konditionale Weltanschauung"Verworn(Verworn, 1912) presented "an exposition of 'conditionism'as contrasted with 'causalism,'(Unknown, 1913) while ignoring cause and effect relationships completely. "**Das Ding ist also identisch mit der Gesamtheit seiner Bedingungen**."(Verworn, 1912) However, Verworn's goal to exterminate causality completely out of science was hindered by the further development of research. The history of futile attempts to refute **the principle of causality** culminated in a publication by the German born physicist Werner Karl Heisenberg (1901 - 1976). Heisenberg put forward an illogical, inconsistent and confusing uncertainty principle which opened the door to wishful thinking and logical fallacies in physics and in science as such. Heisenberg's unjustified reasoning ended in an act of a manifestly unfounded conclusion: "**Weil alle Experimente den Gesetzen der Quantenmechanik und damit der Gleichung (1) unterworfen sind, so wird durch die Quantenmechanik die Ungültigkeit des Kausalgesetzes definitiv festgestellt**."(Heisenberg, Werner Karl, 1927) while 'Gleichung (1)'denotes Heisenberg's uncertainty principle. Einstein's himself, a major contributor to quantum theory and in the same respect a major critic of quantum theory, disliked Heisenberg's uncertainty principle fundamentally while Einstein's opponents used Heisenberg's Uncertainty Principle against Einstein. After the End of the German Nazi initiated Second World War with unimaginable brutality and high human losses and a death toll due to an industrially organised mass killing of people by the German Nazis which did not exist in this way before, Werner Heisenberg visited Einstein in Princeton (New Jersey, USA) in October 1954 (Neffe, 2006). Einstein agreed to meet Heisenberg only for a very short period of time but their encounter lasted longer. However, there where not only a number of differences between Einstein and Heisenberg, these two physicists did not really loved each other. "Einstein remarked that the inventor of the uncertainty principle was a 'big Nazi'... "(Neffe, 2006) Albert Einstein (1879 - 1955) took again the opportunity to refuse to endorse **Heisenberg's uncertainty principle** as a fundamental law of nature and rightly too. Meanwhile, Heisenberg's uncertainty principle is refuted (see Barukčić, 2011a, 2014, 2016a) for several times but still not exterminated completely out of physics and out of science as such. In contrast to such extreme anti-causal positions as advocated by Heisenberg and the **Copenhagen interpretation of quantum mechancis**, the search for a (mathematical) solution of *the issue of causal inferences* is as old as human mankind itself ("*i. e. Aristotle's Doctrine of the Four Causes*") (Hennig, 2009) even if there is still little to go on. It is appropriate to specify especially the position of D'Holbach(Holbach, Paul Henri Thiry Baron de, 1770). D'Holbach (1723-1789) himself linked cause and effect or causality as such to changes. "**Une *cause*, est un être qui e met un autre en mouvement, ou qui produit quelque changement en lui. *L'effet* est le changement qu'un corps produit dans un autre ...**"(Holbach, Paul Henri Thiry Baron de, 1770) D'Holbach infers in the following: "**De l'action et de la réaction continuelle de tous les êtres que la nature renferme, il résulte une suite de causes et d'effets ..**"(Holbach, Paul Henri Thiry Baron de, 1770) With more or less meaningless or none progress on the matter in hand even in the best possible conditions, it is not surprising that authors are suggesting more and more different approaches and models for causal inference. Indeed, the hope is justified that logically consistent *statistical methods of causal inference* can help scientist to achieve so much with so little. One of the methods of causal inference in Bio-sciences are based on the known *Henle(Henle, 1840) (1809–1885) - Koch(Koch, 1878) (1843–1910) postulates* (Carter, 1985) which are applied especially for the identification of a causative agent of an (infectious) disease. However, the pathogenesis of most chronic diseases is more or less very complex and potentially involves the interaction of several factors. In practice, from the 'pure culture' requirement of the Henle-Koch postulates insurmountable difficulties may emerge. In light of subsequent developments (PCR methodology, immune antibodies et cetera) it is appropriate to review the full validity of the Henle-Koch postulates in our days. In 1965, Sir Austin Bradford Hill (Hill, 1965) published nine criteria (the '*Bradford Hill Criteria* ') in order to determine whether observed epidemiologic associations are causal. Somewhat worrying, is at least the fact that, Hill's "... fourth characteristic is *the temporal relationship of the association* " and so-to-speak just a reformulation of the '*post hoc ergo propter hoc'(Barukčić, 1989, Woods and Walton, 1977)* logical fallacy through the back-door and much more then this. It is questionable whether association as such can be treated as being identical with causation. Unfortunately, due to several reasons, it seems therefore rather problematic to rely on Bradford Hill Criteria carelessly. Meanwhile, several other and competing mathematical or statistical approaches for causal inference have been discussed (Barukčić, 1989, 1997, 2005, 2016b, 2017a,c, Bohr, 1937, Dempster, 1990, Espejo, 2007, Hessen, Johannes, 1928, Hesslow, 1976, 1981, Korch, Helmut, 1965, Pearl, 2000, Schlick, Friedrich Albert Moritz, 1931, Suppes, 1970, Zesar, 2013) or even established (Barukčić, 1989, 1997, 2005, 2016b, 2017a,c). Nevertheless, the question is still not answered, is it at all possible to establish a cause effect relationship between two factors while applying only certain statistical (Sober, 2001) methods?

Definition 2.53 (**Causal relationship k**).

Nonetheless, mathematically, the causal(Barukčić, 2011a,b, 2012) relationship (Barukčić, 1989, 1997, 2005, 2016b, 2017a,c, 2021c) between a cause $\mathrm{U_t}$ (German: Ursache) and an effect $\mathrm{W_t}$ (German: Wirkung), denoted by $\mathrm{k(U_t, W_t)}$, is defined *at each single(Thompson, 2006) Bernoulli trial t* in terms of statistics and probability theory as

$$\begin{aligned} k(U_{\mathrm{t}}, W_{\mathrm{t}}) &\equiv \frac{\sigma(U_{\mathrm{t}}, W_{\mathrm{t}})}{\sigma(U_{\mathrm{t}}) \times \sigma(W_{\mathrm{t}})} \\ &\equiv \frac{p(U_{\mathrm{t}} \wedge W_{\mathrm{t}}) - p(U_{\mathrm{t}}) \times p(W_{\mathrm{t}})}{\sqrt[2]{(p(U_{\mathrm{t}}) \times (1 - p(U_{\mathrm{t}}))) \times (p(W_{\mathrm{t}}) \times (1 - p(W_{\mathrm{t}})))}} \end{aligned} \tag{120}$$

where σ ($\mathrm{U_t}$, $\mathrm{W_t}$) denotes the co-variance between a cause $\mathrm{U_t}$ and an effect $\mathrm{W_t}$ *at every single*

Bernoulli trial t, σ (U_t) denotes the standard deviation of a cause U_t at the same single Bernoulli trial t, σ (W_t) denotes the standard deviation of an effect W_t at same single Bernoulli trial t. Table 12 illustrates the theoretically possible relationships between a cause and an effect.

Table 12. Sample space and the causal relationship k

		Effect B_t		
		TRUE	FALSE	
Cause	TRUE	$p(a_t)$	$p(b_t)$	$p(U_t)$
A_t	FALSE	$p(c_t)$	$p(d_t)$	$p(\underline{U}_t)$
		$p(W_t)$	$p(\underline{W}_t)$	+1

However, even if one thinks to recognise the trace of Bravais (Bravais, 1846) (1811-1863) - Pearson's (1857-1936) "*product-moment coefficient of correlation*"(Galton, 1877, Pearson, 1896) inside the causal relationship k (Barukčić, 1989, 1997, 2005, 2016b, 2017a,c) both are completely different. According to Pearson: "*The fundamental theorems of correlation were for the first time and almost exhaustively discussed by B r a v a i s ('Analyse mathematique sur les probabilities des erreurs de situation d'un point.' Memoires par divers Savans, T. IX., Paris, 1846, pp. 255-332) nearly half a century ago.*"(Pearson, 1896) Neither does it make much sense to elaborate once again on the issue causation(Blalock, 1972) and correlation, since both are not identical (Sober, 2001) nor does it make sense to insist on the fact that "*Pearson's philosophy discouraged him from looking too far behind phenomena.*"(Haldane, 1957) Whereas it is essential to consider that the causal relationship k, in contrast to Pearson's product-moment coefficient of correlation(Pearson, 1896) or to Pearson's phi coefficient(Pearson, 1904b), is defined at every single Bernoulli trial t. This might be a very *small* difference. However, even a small difference might determine a difference. However, in this context and in any case, this small difference *makes*(Barukčić, 2018a) the difference.

2.2.22. Cause and effect

Definition 2.54 (Cause and effect).

What is the cause, what is the effect? Under conditions of a **positive** causal relationship k, an event U_t which is for sure a cause of another event W_t is at the same time t a necessary and sufficient condition of an event W_t. Table 13 may illustrate this relationship.

As can be seen, there is a kind of strange mirroring between U_t and W_t at the same Bernoulli trial t. Lastly, both are converses of each other too. In other words, U_t's being a necessary condition of W_t's is equivalent to W_t's being a sufficient condition of U_t's (and vice versa). In general, it is

$$(U_t \vee \underline{W}_t) \equiv (\underline{W}_t \vee U_t) \equiv ((U_t \vee \underline{W}_t) \wedge (\underline{W}_t \vee U_t)) \equiv +1 \tag{121}$$

In our everyday words,

Table 13. What is the cause, what is the effect?

		Effect W_t		
		TRUE	FALSE	
Cause	TRUE	**+1**	**+0**	$p(U_t)$
U_t	FALSE	**+0**	**+1**	$p(\underline{U}_t)$
		$p(W_t)$	$p(\underline{W}_t)$	+1

without

U_t

no

W_t

is equivalent with

if

W_t

then

U_t

and vice versa.

Necessary and sufficient conditions are relationships used to describe the relationship between two events at the same Bernoulli trial t. In more detail, if U_t then W_t is equivalent with W_t is necessary for U_t, because the truth of U_t guarantees the truth of W_t. In general, it is

$$(\underline{U}_t \vee W_t) \equiv (W_t \vee \underline{U}_t) \equiv ((\underline{U}_t \vee W_t) \wedge (W_t \vee \underline{U}_t)) \equiv +1 \tag{122}$$

In other words, it is impossible to have U_t without W_t (Bloch, 2011). Similarly, U_t is sufficient for W_t, because U_t being true always implies that W_t is true, but U_t not being true does not always imply that W_t is not true.

For instance, **without** gaseous oxygen (U_t), there would be **no** burning wax candle (W_t); hence the relationship **if** burning wax candle (W_t) **then** gaseous oxygen (U_t) is equally true and given.

This simple example may illustrate the reason why a sufficient condition alone is not enough to describe a cause completely. The relationship **if** burning wax candle (W_t) **then** gaseous oxygen (U_t) is given. Independently of this fact, a burning wax candle is not the cause of gaseous oxygen. Therefore, in order to be a cause of oxygen, additional evidence is necessary that a burning wax candle is a necessary condition of gaseous oxygen too. However, even if the relationship **without** gaseous oxygen **no** burning wax candle is given, this relationship is not given vice versa. The relationship **without** burning wax candle **no** gaseous oxygen is not given. Like other fundamental concepts, the concepts of cause and effect can be associated with difficulties too. In order to recognise a causal relationship between U_t and W_t, it is necessary that the same study or that at least different studies provide evidence of a necessary condition between U_t and W_t and of a sufficient condition between U_t and W_t and if possible of **a necessary and sufficient condition** between U_t and W_t too.

Mathematically, a necessary and sufficient condition between U_t and W_t is defined as

$$(U_{\mathrm{t}} \vee \underline{W}_{\mathrm{t}}) \wedge (\underline{U}_{\mathrm{t}} \vee W_{\mathrm{t}}) \equiv +1 \tag{123}$$

However, I think it necessary to make a clear distinction between a necessary and sufficient condition and the converse relationship (Eq. 121) above.

$$((U_{\mathrm{t}} \vee \underline{W}_{\mathrm{t}}) \wedge (\underline{W}_{\mathrm{t}} \vee U_{\mathrm{t}})) \neq (U_{\mathrm{t}} \vee \underline{W}_{\mathrm{t}}) \wedge (\underline{U}_{\mathrm{t}} \vee W_{\mathrm{t}}) \tag{124}$$

2.3. *Axioms*

2.3.1. Axiom I. Lex identitatis

In this context, we define axiom I as the expression

$$+1 = +1 \tag{125}$$

2.3.2. Axiom II. Lex contradictionis

In this context, axiom II or **lex contradictionis**, the negative of lex identitatis, or

$$+0 = +1 \tag{126}$$

and equally the most simple form of a contradiction formulated.

2.3.3. Axiom III. Lex negationis

$$\neg(0) \times 0 = 1 \tag{127}$$

where $\neg$ denotes (logical (Boole, 1854) or natural) negation (Ayer, 1952, Förster and Melamed, 2012, Hedwig, 1980, Heinemann, Fritz H., 1943, Horn, 1989, Koch, 1999, Kunen, 1987, Newstadt, 2015, Royce, 1917, Speranza and Horn, 2010, Wedin, 1990). In this context, there is some evidence that $\neg(1) \times 1 = 0$. In other words, it is $(\neg(1) \times 1) \times (\neg(0) \times 0) = 1$

3. Results

3.1. Approximate value of a sufficient condition relationship I

Theorem 3.1 (Approximate value of a sufficient condition relationship I)**.** *In general, it is*

$$p(A_t \rightarrow B_t) \geq 1 - \frac{p(b_t)}{p(A_t)} \tag{128}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{129}$$

is true. Equation 129 (the premise) is rearranged. We obtain

$$p(b_{\mathrm{t}}) \equiv p(b_{\mathrm{t}}) \tag{130}$$

Under conditions of independence, equation 130 (see equation 62, p. 23) becomes

$$p(b_{\mathrm{t}}) \equiv p(A_{\mathrm{t}}) \times p(\underline{B}_{\mathrm{t}}) \tag{131}$$

Rearranging equation 131 , it is

$$\frac{p(b_{\mathrm{t}})}{p(A_{\mathrm{t}})} \equiv p(\underline{B}_{\mathrm{t}}) \equiv 1 - p(B_{\mathrm{t}}) \tag{132}$$

or

$$1 - \frac{p(b_{\mathrm{t}})}{p(A_{\mathrm{t}})} \equiv p(B_{\mathrm{t}}) \tag{133}$$

The conditio per quam relationship follows as

$$p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \equiv 1 - \frac{p(b_{\mathrm{t}})}{p(A_{\mathrm{t}})} + p(d_{\mathrm{t}}) \equiv p(B_{\mathrm{t}}) + p(d_{\mathrm{t}}) \tag{134}$$

While ignoring the value of $p(d_{\mathrm{t}})$, the approximate value of the material implication follows as

$$p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \geq 1 - \frac{p(b_{\mathrm{t}})}{p(A_{\mathrm{t}})} \tag{135}$$

□

3.2. Approximate value of a sufficient condition relationship II

Theorem 3.2 (Approximate value of a sufficient condition relationship II)**.** *In general, it is*

$$p(A_t \rightarrow B_t) \geq 1 - \frac{p(b_t)}{p(\underline{B}_t)} \tag{136}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{137}$$

is true. Equation 137 (the premise) is rearranged. We obtain

$$p(b_t) \equiv p(b_t) \tag{138}$$

Under conditions of independence, equation 138 (see equation 62, p. 23) becomes

$$p(b_t) \equiv p(A_t) \times p(\underline{B}_t) \tag{139}$$

Rearranging equation 139 , it is

$$\frac{p(b_t)}{p(\underline{B}_t)} \equiv p(A_t) \equiv 1 - p(\underline{A}_t) \tag{140}$$

or

$$1 - \frac{p(b_t)}{p(\underline{B}_t)} \equiv p(\underline{A}_t) \tag{141}$$

The conditio per quam relationship follows as

$$p(A_t \rightarrow B_t) \equiv 1 - \frac{p(b_t)}{p(\underline{B}_t)} + p(a_t) \equiv p(\underline{A}_t) + p(a_t) \tag{142}$$

While ignoring the value of $p(a_t)$, the approximate value of the material implication follows as

$$p(A_t \rightarrow B_t) \geq 1 - \frac{p(b_t)}{p(\underline{B}_t)} \tag{143}$$

□

3.3. Experimental studies and sufficient conditions

Experimental [3] , [4] trials and observational studies are the two main approaches to medical investigations. Mathematically, it is possible to estimate the extent to which an event A_t is a sufficient condition of an event B_t (an outcome) independent of a placebo group.

Theorem 3.3. *In general, the sufficient condition relationship follows approximately as*

$$p(A_t \rightarrow B_t) \geq 1 - \frac{p(b_t)}{p(A_t)} \tag{144}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{145}$$

[3] Gjorgov AN. Experimental studies: randomized clinical trials. Folia Med (Plovdiv). 1998;40(3B Suppl 3):9-16. PMID: 10205986.

[4] Moorhead JE, Rao PV, Anusavice KJ. Guidelines for experimental studies. Dent Mater. 1994 Jan;10(1):45-51. doi: 10.1016/0109-5641(94)90021-3. PMID: 7995475.

is true. In the following, we rearrange the premise. We obtain

$$p(A_t) \equiv p(A_t) \tag{146}$$

or

$$p(a_t) + p(b_t) \equiv p(A_t) \tag{147}$$

Rearranging equation 147, it is

$$p(a_t) \equiv p(A_t) - p(b_t) \tag{148}$$

Simplifying equation 148, we obtain

$$\frac{p(a_t)}{p(A_t)} \equiv \frac{p(A_t)}{p(A_t)} - \frac{p(b_t)}{p(A_t)} \tag{149}$$

Equation 149 becomes

$$\frac{p(a_t)}{p(A_t)} \equiv 1 - \frac{p(b_t)}{p(A_t)} \tag{150}$$

A basic requirement of a sufficient condition relationship is the need that $\frac{p(a_t)}{p(A_t)} \equiv 1$. In general, it is

$$p(A_t \rightarrow B_t) \equiv \frac{p(a_t)}{p(A_t)} \equiv 1 - \frac{p(b_t)}{p(A_t)} \tag{151}$$

However, this relationship is not given under any circumstances. Therefore, the sufficient condition relationship can be estimated roughly under conditions of an experimental study independently of a control group by the relationship

$$p(A_t \rightarrow B_t) \approx 1 - \frac{p(b_t)}{p(A_t)} \tag{152}$$

□

However, in reality, it can be assumed that the sufficient condition relationship will be stronger than the relationship suggested by equation 152. Therefore, equation 152 is of particular value under conditions where a placebo group is absent or appears to be (completely) unsuitable. In general, it is

$$p(A_t \rightarrow B_t) \geq 1 - \frac{p(b_t)}{p(A_t)} \tag{153}$$

3.4. Observational studies and sufficient conditions

Comparing different events under different conditions [5] , [6] of observational studies (case report or case series, ecologic, cross-sectional, cohort, case-control, nested case-control, and case-cohort) can lead to new insights. Mathematically, it is possible to estimate the extent to which an event A_t is a sufficient condition of an event B_t (an outcome).

[5] Hoffmann RG, Lim HJ. Observational study design. Methods Mol Biol. 2007;404:19-31. doi: 10.1007/978-1-59745-530-5_2. PMID: 18450043.

[6] DiPietro NA. Methods in epidemiology: observational study designs. Pharmacotherapy. 2010 Oct;30(10):973-84. doi: 10.1592/phco.30.10.973. PMID: 20874034.

Theorem 3.4. *In general, the sufficient condition relationship follows approximately as*

$$p(A_t \rightarrow B_t) \geq 1 - \frac{p(b_t)}{p(\underline{B}_t)} \tag{154}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{155}$$

is true. In the following, we rearrange the premise. We obtain

$$p(\underline{B}_{\mathrm{t}}) \equiv p(\underline{B}_{\mathrm{t}}) \tag{156}$$

or

$$p(b_{\mathrm{t}}) + p(d_{\mathrm{t}}) \equiv p(\underline{B}_{\mathrm{t}}) \tag{157}$$

Rearranging equation 157, it is

$$p(d_{\mathrm{t}}) \equiv p(\underline{B}_{\mathrm{t}}) - p(b_{\mathrm{t}}) \tag{158}$$

Simplifying equation 158, we obtain

$$\frac{p(d_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \equiv \frac{p(\underline{B}_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} - \frac{p(b_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \tag{159}$$

Equation 159 becomes

$$\frac{p(d_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \equiv 1 - \frac{p(b_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \tag{160}$$

However, another basic requirement of a sufficient condition relationship is the need that $\frac{p(d_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \equiv 1$. In general, it is

$$p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \equiv \frac{p(d_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \equiv 1 - \frac{p(b_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \tag{161}$$

Regrettably, this reduced relationship of a sufficient condition is not given under any circumstances too. In other words, the sufficient condition relationship can be estimated roughly under conditions of an observational study by the relationship

$$p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \approx 1 - \frac{p(b_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \tag{162}$$

□

However, in reality, it can be assumed that the sufficient condition relationship will be stronger than the relationship suggested by equation 162. Therefore, equation 162 is of particular value under conditions where suitable data are absent or (completely) inappropriate, et cetera. In general, it is

$$p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \geq 1 - \frac{p(b_{\mathrm{t}})}{p(\underline{B}_{\mathrm{t}})} \tag{163}$$

3.5. Study design and sufficient conditions

The study design of an observational or an experimental study should assure that it should be possible to recognize a sufficient condition given, it doesn't matter whether data are obtained by an observational or an experimental study. What is a basic requirement of such a study design?

Theorem 3.5. *In general, the sufficient condition relationship demands a study design where the index of unfairness (IOU) (Barukčić, 2019c) or p(IOU) is equal to*

$$p\left(IOU\left(A,B\right)\right) \equiv Absolute\left(\left(\frac{A_t + B_t}{N}\right) - 1\right) \equiv 0 \tag{164}$$

Proof by direct proof. The premise

$$+1 \equiv +1 \tag{165}$$

is true. In the following, we rearrange the premise. We obtain

$$p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right) \equiv p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right) \tag{166}$$

Based on equation 151 it is $p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right) \equiv \frac{p\left(a_{\mathrm{t}}\right)}{p\left(A_{\mathrm{t}}\right)} \equiv 1 - \frac{p\left(b_{\mathrm{t}}\right)}{p\left(A_{\mathrm{t}}\right)}$. Rearranging equation 166, it is

$$1 - \frac{p\left(b_{\mathrm{t}}\right)}{p\left(A_{\mathrm{t}}\right)} \equiv p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right) \tag{167}$$

Based on equation 161 it is $p\left(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}\right) \equiv \frac{p\left(d_{\mathrm{t}}\right)}{p\left(\underline{B}_{\mathrm{t}}\right)} \equiv 1 - \frac{p\left(b_{\mathrm{t}}\right)}{p\left(\underline{B}_{\mathrm{t}}\right)}$. Equation 167 simplifies as

$$1 - \frac{p\left(b_{\mathrm{t}}\right)}{p\left(A_{\mathrm{t}}\right)} \equiv 1 - \frac{p\left(b_{\mathrm{t}}\right)}{p\left(\underline{B}_{\mathrm{t}}\right)} \tag{168}$$

Equation 168 becomes

$$-\frac{p\left(b_{\mathrm{t}}\right)}{p\left(A_{\mathrm{t}}\right)} \equiv -\frac{p\left(b_{\mathrm{t}}\right)}{p\left(\underline{B}_{\mathrm{t}}\right)} \tag{169}$$

or

$$\frac{p\left(b_{\mathrm{t}}\right)}{p\left(\underline{B}_{\mathrm{t}}\right)} \equiv \frac{p\left(b_{\mathrm{t}}\right)}{p\left(A_{\mathrm{t}}\right)} \tag{170}$$

Equation 170 can be simplified as

$$p\left(b_{\mathrm{t}}\right) \times p\left(A_{\mathrm{t}}\right) \equiv p\left(b_{\mathrm{t}}\right) \times p\left(\underline{B}_{\mathrm{t}}\right) \tag{171}$$

In the following we ignore $p\left(b_{\mathrm{t}}\right)$ and set $p\left(b_{\mathrm{t}}\right) \equiv 0.00001$. Under the assumptions above, the study design should ensure as much as possible the relationship

$$p\left(A_{\mathrm{t}}\right) \equiv p\left(\underline{B}_{\mathrm{t}}\right) \tag{172}$$

or

$$p\left(A_{\mathrm{t}}\right) \equiv 1 - p\left(B_{\mathrm{t}}\right) \tag{173}$$

or

$$p(A_{\mathrm{t}}) + p(B_{\mathrm{t}}) \equiv +1 \tag{174}$$

Rearranging equation 174, it is

$$N \times p(A_{\mathrm{t}}) + N \times p(B_{\mathrm{t}}) \equiv N \tag{175}$$

while N might denote the sample or population size. Furthermore, it follows that

$$A_{\mathrm{t}} + B_{\mathrm{t}} \equiv N \tag{176}$$

Rearranging equation 176, it is

$$\frac{A_{\mathrm{t}} + B_{\mathrm{t}}}{N} \equiv \frac{N}{N} \equiv +1 \tag{177}$$

and the index of unfairness (Barukčić, 2019c) (IOU) follows as

$$IOU(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \left(\frac{A_{\mathrm{t}} + B_{\mathrm{t}}}{N}\right) - 1 \equiv 0 \tag{178}$$

In order to ensure that the results of observational and experimental studies obtained which investigated the sufficient condition relationship are comparable with each other, the study design should assure as much as possible that

$$p(IOU(A,B)) \equiv Absolute\left(\left(\frac{A_{\mathrm{t}} + B_{\mathrm{t}}}{N}\right) - 1\right) \equiv 0 \tag{179}$$

□

3.6. Conditio per quam and compound conditions

Causal knowledge of objective reality is an integral part of science. However, the usefulness of causal notions in science is still not generally accepted.

> "The law of causality ... is a relict of bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm. "
>
> (see Russell, 1919)

Especially the conflict between Einstein's relativity [7] , [8] , [9] theory of space and time and (neo-) Kantian [10] a priorism has been a philosophical background of Reichenbach's Common Cause Principle (RCCP) which has been introduced by Hans Reichenbach [11](1891–1953) in his book [12] The Direction of Time (see Reichenbach, 1971), published posthumously in 1956. Unfortunately, a number of counterexamples (see Cartwright, 1999, p. 108–109) have been proposed with respect to RCCP. Meanwhile, Reichenbach's Common Cause Principle (see Hofer-Szabó et al., 1999) is closely related to the highly controversial (see Cartwright, 2002) Causal Markov Condition (see Martel, 2008). It's a matter of common knowledge that, with respect to the generation of scientific knowledge, a different kind of bias [13] is present to some degree in all research. A researcher might attempt to investigate the relationship between an exposure (A_t) and an outcome (B_t) but does not consider the effect of third factors (the confounding [14] variable) to a necessary degree. Even randomised clinical trials [15] does not exclude systematic bias and erroneous results completely. Multiple events (conditions, i.e. ${}_1X_t$, ${}_2X_t$, ${}_3X_t$, $\cdots$) at the same (period of) time t / Bernoulli trial t can be combined with the condition A_t into a compound condition with the help of the logical or Boolean operators, AND, OR, and a third operator, NOT. However, this has no influence on the material implications or the conditio per quam relationship.

[7] Ryckman, Thomas A., "Early Philosophical Interpretations of General Relativity", The Stanford Encyclopedia of Philosophy (Fall 2021 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/fall2021/entries/genrel-early/.

[8] Will CM. The Confrontation between General Relativity and Experiment. Living Rev Relativ. 2014;17(1):4. doi: 10.12942/lrr-2014-4. Epub 2014 Jun 11. PMID: 28179848; PMCID: PMC5255900.

[9] Rosen SM. Why natural science needs phenomenological philosophy. Prog Biophys Mol Biol. 2015 Dec;119(3):257-69. doi: 10.1016/j.pbiomolbio.2015.06.008. Epub 2015 Jul 2. PMID: 26143599.

[10] Heis, Jeremy, "Neo-Kantianism", The Stanford Encyclopedia of Philosophy (Summer 2018 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/sum2018/entries/neo-kantianism/.

[11] Glymour, Clark and Frederick Eberhardt, "Hans Reichenbach", The Stanford Encyclopedia of Philosophy (Summer 2021 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/sum2021/entries/reichenbach/.

[12] Hitchcock, Christopher and Miklós Rédei, "Reichenbach's Common Cause Principle", The Stanford Encyclopedia of Philosophy (Summer 2021 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/sum2021/entries/physics-Rpcc/.

[13] Hammer GP, du Prel JB, Blettner M. Avoiding bias in observational studies: part 8 in a series of articles on evaluation of scientific publications. Dtsch Arztebl Int. 2009 Oct;106(41):664-8. doi: 10.3238/arztebl.2009.0664. Epub 2009 Oct 9. PMID: 19946431; PMCID: PMC2780010.

[14] Grimes DA, Schulz KF. Bias and causal associations in observational research. Lancet. 2002 Jan 19;359(9302):248-52. doi: 10.1016/S0140-6736(02)07451-2. PMID: 11812579.

[15] Atkins D, Eccles M, Flottorp S, Guyatt GH, Henry D, Hill S, Liberati A, O'Connell D, Oxman AD, Phillips B, Schünemann H, Edejer TT, Vist GE, Williams JW Jr; GRADE Working Group. Systems for grading the quality of evidence and the strength of recommendations I: critical appraisal of existing approaches The GRADE Working Group. BMC Health Serv Res. 2004 Dec 22;4(1):38. doi: 10.1186/1472-6963-4-38. PMID: 15615589; PMCID: PMC545647.

One mathematical formula of a compound condition relationship is given as

$$
\begin{aligned}
p\left(\left(\left({}_{1}X_{\mathrm{t}} \wedge {}_{2}X_{\mathrm{t}} \wedge {}_{3}X_{\mathrm{t}} \wedge \cdots\right) \wedge A_{\mathrm{t}}\right) \rightarrow B_{\mathrm{t}}\right) &\equiv p\left(\underline{\left(\left({}_{1}X_{\mathrm{t}} \wedge {}_{2}X_{\mathrm{t}} \wedge {}_{3}X_{\mathrm{t}} \wedge \cdots\right) \wedge A_{\mathrm{t}}\right)} \vee B_{\mathrm{t}}\right) \\
&\equiv \frac{\sum_{t=1}^{N}\left(\underline{\left(\left({}_{1}X_{\mathrm{t}} \wedge {}_{2}X_{\mathrm{t}} \wedge {}_{3}X_{\mathrm{t}} \wedge \cdots\right) \wedge A_{\mathrm{t}}\right)} \vee B_{\mathrm{t}}\right)}{N} \\
&\equiv +1
\end{aligned}
\tag{180}
$$

So far, the counterfactual theories works under the assumption that if $\left(\left({}_{1}X_{\mathrm{t}} \wedge {}_{2}X_{\mathrm{t}} \wedge {}_{3}X_{\mathrm{t}} \wedge \cdots\right) \wedge A_{\mathrm{t}}\right)$ had not [16] occurred, B_{t} would not have occurred (see Lewis, 1974) too, which is not of very great value for conditio per quam relationship. In the hope of casting some light on the issue of compound sufficient [17], [18] conditions, let us present this relationship once again by the table 14 in more detail.

Table 14. Compound condition and material implication.

Bernoulli trial t	$\left({}_{1}X_{\mathrm{t}} \wedge {}_{2}X_{\mathrm{t}} \wedge {}_{3}X_{\mathrm{t}} \wedge \cdots\right) \wedge A_{\mathrm{t}}$	B_{t}	$p\left(\left({}_{1}X_{\mathrm{t}} \wedge {}_{2}X_{\mathrm{t}} \wedge {}_{3}X_{\mathrm{t}} \wedge \cdots \wedge A_{\mathrm{t}}\right) \rightarrow B_{\mathrm{t}}\right)$
1	1	1	1
2	1	0	0
3	0	1	1
4	0	0	1
...	...	...	...

Example.

Often, drivers are killed or critically injured in a car crash. However, not every car crash (A_{t}) ends up in a deadly event (B_{t}). In other words, the hypothesis, **if** car crash **then** deadly event is not correct. However, additional conditions like $\left(\left({}_{1}X_{\mathrm{t}} \wedge {}_{2}X_{\mathrm{t}} \wedge {}_{3}X_{\mathrm{t}} \wedge \cdots\right)\right)$ are sometimes necessary for a car crash being a sufficient condition of a deadly event.

[16] Menzies, Peter and Helen Beebee, "Counterfactual Theories of Causation", The Stanford Encyclopedia of Philosophy (Winter 2020 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2020/entries/causation-counterfactual/.

[17] López-Astorga M, Ragni M, Johnson-Laird PN. The probability of conditionals: A review. Psychon Bull Rev. 2022 Feb;29(1):1-20. doi: 10.3758/s13423-021-01938-5. Epub 2021 Jun 25. PMID: 34173186; PMCID: PMC8231749.

[18] Over DE, Hadjichristidis C, Evans JS, Handley SJ, Sloman SA. The probability of causal conditionals. Cogn Psychol. 2007 Feb;54(1):62-97. doi: 10.1016/j.cogpsych.2006.05.002. Epub 2006 Jul 12. PMID: 16839539.

3.7. Material implication and risk ratio

The relative (Cornfield, 1951, Sadowsky et al., 1953) risk or risk ratio is one of the proposed concepts and methods in bio-statistics which is useful [19], [20] under certain particular circumstances. However, the possibility of the risk ratio

$$RR_{\mathrm{nc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{a \times \underline{A}}{c \times A} \tag{181}$$

to recognise a material implication is restricted. The need for a new, logically consistent and systematic approach to address this issue is essential.

Theorem 3.6 (**MATERIAL IMPLICATION AND RISK RATIO**).

CLAIM.
In general, under some circumstances the risk ratio, denoted as $\mathrm{RR_{sc}}$ ($\mathrm{A_t}$, $\mathrm{B_t}$), defined as

$$RR_{\mathrm{sc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{a \times \underline{B}}{b \times B} \tag{182}$$

provides only an approximate and equally imprecise picture of a material condition (see Barukčić, 2021d).

Proof by direct proof. The premise [21] is

$$+1 \equiv +1 \tag{183}$$

Consequently if this premise is true, then the conclusion is also true, the absence of any technical and other errors of human reasoning assumed. Nonetheless, the premise is true. Multiplying the premise (i. e. eq. 183) by $(p(A_{\mathrm{t}}) \times p(B_{\mathrm{t}}))$ it is

$$p(A_{\mathrm{t}}) \times p(B_{\mathrm{t}}) \equiv p(A_{\mathrm{t}}) \times p(B_{\mathrm{t}}) \tag{184}$$

Under conditions of probability theory and in the case of independence of both events $\mathrm{A_t}$ and $\mathrm{B_t}$ at a certain (period of) time / Bernoulli (see also Uspensky, 1937) trial t it is according to de Moivre [22] and Kolmogoroff [23] and other

$$p(a_{\mathrm{t}}) \equiv p(A_{\mathrm{t}} \wedge B_{\mathrm{t}}) \equiv p(A_{\mathrm{t}}) \times p(B_{\mathrm{t}}) \tag{185}$$

[19] Vandenbroucke JP, Broadbent A, Pearce N. Causality and causal inference in epidemiology: the need for a pluralistic approach. Int J Epidemiol. 2016 Dec 1;45(6):1776-1786. doi: 10.1093/ije/dyv341. PMID: 26800751; PMCID: PMC5841832.
[20] Rothman KJ, Greenland S. Causation and causal inference in epidemiology. Am J Public Health. 2005;95 Suppl 1:S144-50. doi: 10.2105/AJPH.2004.059204. PMID: 16030331.
[21] http://www.ijmttjournal.org/archive/ijmtt-v65i7p524
[22] https://doi.org/10.3931/e-rara-10420
[23] https://doi.org/10.1007/978-3-642-49888-6

According to eq. 23 it is $p(A_t) \equiv (p(a_t) + p(b_t))$. Eq. 185 changes too

$$p(a_t) \equiv (p(a_t) + p(b_t)) \times p(B_t) \tag{186}$$

or too

$$p(a_t) \equiv (p(a_t) \times p(B_t)) + (p(b_t) \times p(B_t)) \tag{187}$$

Rearranging eq. 187 it is

$$p(a_t) - (p(a_t) \times p(B_t)) \equiv p(b_t) \times p(B_t) \tag{188}$$

or

$$p(a_t) \times (1 - p(B_t)) \equiv p(b_t) \times p(B_t) \tag{189}$$

Based on eq. 32 it is $p(\underline{B}_t) \equiv (1 - p(B_t))$. **Under conditions of independence eq. 189 changes further**. In general, is is necessary to accept that

$$p(a_t) \times p(\underline{B}_t) \equiv p(b_t) \times p(B_t) \tag{190}$$

Under conditions of independence eq. 190 implies too that

$$\frac{p(a_t)}{p(B_t)} \equiv \frac{X \times p(a_t)}{X \times p(B_t)} \equiv \frac{p(b_t)}{p(\underline{B}_t)} \tag{191}$$

Eq. 191 can be tested by a kind of a Chi-square goodness of fit test as $\tilde{\chi}^2{}_{\text{Calculated}} \equiv N \times \sum_{t=1}^{t=N} \left(\frac{\left(\frac{p(a_t)}{p(B_t)} - \frac{p(b_t)}{p(\underline{B}_t)} \right)^2}{\frac{p(b_t)}{p(\underline{B}_t)}} \right)$, a sum of differences between the observed and the expected. From 191 follows too that

$$p(a_t) \equiv p(b_t) \times \frac{p(B_t)}{p(\underline{B}_t)} \tag{192}$$

or that

$$p(b_t) \equiv p(a_t) \times \frac{p(\underline{B}_t)}{p(B_t)} \tag{193}$$

However, eq. 190 derived as $p(a_t) \times p(\underline{B}_t) \equiv p(b_t) \times p(B_t)$ can be rearranged further as

$$\frac{p(a_t) \times p(\underline{B}_t)}{p(b_t) \times p(B_t)} \equiv +1 \tag{194}$$

which is a very approximate and equally a very imprecise picture of a sufficient condition provided to us by the risk ratio RR_{sc} (A_t, B_t) as

$$RR_{\mathrm{sc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{p(a_{\mathrm{t}}) \times p(\underline{B_{\mathrm{t}}})}{p(b_{\mathrm{t}}) \times p(B_{\mathrm{t}})} \equiv +1 \tag{195}$$

Under conditions where each trial is independent of another trial and where the probability of an event is constant from trial to trial it is equally

$$RR_{\mathrm{sc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{p(a_{\mathrm{t}}) \times p(\underline{B_{\mathrm{t}}})}{p(b_{\mathrm{t}}) \times p(B_{\mathrm{t}})} \equiv \frac{N^2 \times p(a_{\mathrm{t}}) \times p(\underline{B_{\mathrm{t}}})}{N^2 \times p(b_{\mathrm{t}}) \times p(B_{\mathrm{t}})} \equiv \frac{a \times \underline{B}}{b \times B} \equiv +1 \tag{196}$$

where a, b, B (i. e. cases) and B̲ (i. e. controls) may denote the expectation values and our conclusion is true.

□

Obviously, there is a relationship between $\mathrm{RR_{sc}(A_t,\ B_t)}$ and material implication. From equation 196

$$RR_{\mathrm{sc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{a \times \underline{B}}{b \times B} \tag{197}$$

follows $(itis: a_{\mathrm{t}} = (N \times p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) - A_{\mathrm{t}}))$ that

$$RR_{\mathrm{sc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{((N \times p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}})) - A_{\mathrm{t}}) \times \underline{B}}{b \times B} \tag{198}$$

and equally that

$$p(A_{\mathrm{t}} \rightarrow B_{\mathrm{t}}) \equiv \frac{\frac{RR_{\mathrm{sc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \times b_{\mathrm{t}} \times B_{\mathrm{t}}}{\underline{B_{\mathrm{t}}}} + A_{\mathrm{t}}}{N} \tag{199}$$

In other words, the risk ratio $RR_{\mathrm{sc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{p(a_{\mathrm{t}}) \times p(\underline{B_{\mathrm{t}}})}{p(b_{\mathrm{t}}) \times p(B_{\mathrm{t}})} \equiv \frac{N^2 \times p(a_{\mathrm{t}}) \times p(\underline{B_{\mathrm{t}}})}{N^2 \times p(b_{\mathrm{t}}) \times p(B_{\mathrm{t}})} \equiv \frac{a \times \underline{B}}{b \times B} > +1$ provides some, even if very slight and approximate evidence, that $\mathrm{A_t}$ is a sufficient condition of $\mathrm{B_t}$. However, it makes much more sense to use the original(see also Barukčić, 2021c) sufficient[24] condition formula. In the same way, the relationship between $RR_{\mathrm{nc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \equiv \frac{a \times \underline{A}}{c \times A}$ and necessary condition $p(A_{\mathrm{t}} \leftarrow B_{\mathrm{t}}) \equiv \frac{\frac{RR_{\mathrm{nc}}(A_{\mathrm{t}}, B_{\mathrm{t}}) \times c_{\mathrm{t}} \times A_{\mathrm{t}}}{\underline{A_{\mathrm{t}}}} + \underline{B_{\mathrm{t}}}}{N}$ can be established.

[24]https://www.matec-conferences.org/articles/matecconf/abs/2021/05/matecconf_cscns20_09032/matecconf_cscns20_09032.html

3.8. Material implication and causation

In discussing the role of causation in science, scientist disagree even on fundamental issues of causation and whether there is causation at all. While such anti-causal tradition is perhaps not as dominant today than it once was, there continues to be a active scientific debate even on the basic of causation. In general, under conditions of a deterministic causal relation, a cause at a certain (period of) time t / point in space-time t / Bernoulli trial t should produce its own effect with certainty. In other words, the demand **if cause$_t$ then effect$_t$** is justified. As a consequence, the relationship: if A_t then B_t is often mismatched with causation. However, it is nonetheless worth noting that this need not to be valid in general. A material implication need not proof a cause effect relationship for sure. A counter-example might provide us with the evidence needed. In this context, it is necessary to emphasise once again that **one single counter example**(Bağçe, Samet and Başkent, Can, 2009, Corcoran, 2005, Israël, Hans, 2011, McGee, 1985, Robertson, 1997, Romano and Siegel, 1986, Stoyanov, 2013, Weatherson, 2003) has the potential to refute a theorem, a theory, a conjecture **as effectively as n** counter examples.

> **"No amount of experimentation**
> **can ever prove me right;**
> **a single experiment**
> **can prove me wrong."**
> (Albert Einstein according to Robertson, 1997)

Theorem 3.7 (Material implication and causation)**.** *In general, material implication is not identical with causation.*

Proof by counter-example. For instance, it is generally known that **without** sufficient amounts of gaseous oxygen (A_t), there would be **no** burning wax candle (B_t). As a matter of fact, the relationship, **without** gaseous oxygen (A_t) **no** burning wax candle (B_t) is true. Table 15 might illustrate this relationship in more detail.

Table 15. Without gaseous oxygen no burning candle.

		Burning candle B_t		
		YES	NO	
Gaseous oxygen	YES	**+1**	**+1**	$p(A_t)$
A_t	NO	**+0**	**+1**	$p(\underline{A}_t)$
		$p(B_t)$	$p(\underline{B}_t)$	+1

As found before, there are circumstances where a necessary and sufficient condition are converses of each other(see equation 102, p. 40). It is

$$p\,(A_t \leftarrow B_t) \equiv (A_t \vee \underline{B}_t) \equiv (\underline{B}_t \vee A_t) \equiv p\,(B_t \rightarrow A_t) \tag{200}$$

In other words, the relationship **if** burning wax candle (B_t) **then** gaseous oxygen (A_t) is equally true (see table 16) and given.

Table 16. If burning candle then gaseous oxygen.

		Gaseous oxygen A_t		
		YES	NO	
Burning candle	YES	**+1**	**+0**	$p(B_t)$
B_t	NO	**+1**	**+1**	$p(\underline{B}_t)$
		$p(A_t)$	$p(\underline{A}_t)$	+1

However, even if the conditio per quam relationship or material implication **if** burning candle **then** gaseous oxygen is given for sure, a burning wax candle is neither a cause nor the cause of gaseous oxygen. This simple and reproduceable counter-example provides striking evidence that a material implication is not identical with causation and vice versa. Causation as such cannot be reduced to simple material implication.

□

3.9. Modus ponens and material implication

There can be no doubt that the origins of the development of logic remains mostly in the dark. However, it is justified to assume that logic as a human enterprise is determined especially by human development and human experience too. By time, those human beings who were neither willing nor able to recognise changes of objective reality around them have suffered damage, potentially failed to reproduce themselves or not to a necessary extent and are more or less extinct. Nonetheless, in Greek and Roman antiquity, logic as a fully systematic discipline begins at least with Aristotle. Meanwhile, several rules of inference [25], [26] like modus ponens and modus tollens (Popper, Karl Raimund, 1935) emerged. Following Sir Karl Raimund Popper [27] (1902 -1994)

> "... it is possible by means of purely deductive inferences (with the help of the modus tollens of classical logic) to argue from the truth of singular statements to the falsity of universal statements. "
>
> (Popper, Karl Raimund, 1935, p. 19)

In Popper's own words, there is a scientific method, "... in which the falsification of a conclusion entails the falsification of the system from which it is derived "(Popper, Karl Raimund, 1935, p. 55).

Theorem 3.8 (The modus ponens rule of inference)**.** *Let +1 denote true, let +0 denote false. Let P_t denote an event (the premise) at the (period of) time t / space-time t / the Bernoulli trial t, let C_t denote another event (the conclusion) at the same (period of) time t / space-time t / Bernoulli trial t. Usually, P_t is called the antecedent and C_t the consequent. It is*

$$C_t \equiv +1 \tag{201}$$

Proof by modus ponens. Our premise is that

$$+1 \equiv +1 \tag{202}$$

is true or that the material implication

$$p\left(P_{\mathrm{t}} \rightarrow C_{\mathrm{t}}\right) \equiv +1 \tag{203}$$

is true. Table 17 might illustrate the relationship between modus ponens [28] and material implication once again.

[25] Edgington, Dorothy, "Indicative Conditionals", The Stanford Encyclopedia of Philosophy (Fall 2020 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/fall2020/entries/conditionals/.

[26] Bobzien, Susanne, "Ancient Logic", The Stanford Encyclopedia of Philosophy (Summer 2020 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/sum2020/entries/logic-ancient/.

[27] Thornton, Stephen, "Karl Popper", The Stanford Encyclopedia of Philosophy (Fall 2021 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/fall2021/entries/popper/.

[28] Egré, Paul and Hans Rott, "The Logic of Conditionals", The Stanford Encyclopedia of Philosophy (Winter 2021 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/win2021/entries/logic-conditionals/.

Table 17. Modus ponens and material implication.

		Conclusion C_t		
		YES	NO	
Premise	YES	**+1**	**+0**	$p(P_t)$
P_t	NO	**+1**	**+1**	$p(\underline{P}_t)$
		$p(C_t)$	$p(\underline{C}_t)$	+1

However, modus ponens and material implication are deeply interrelated but not completely identical. It is self-evident and of great importance that a proof by modus ponens itself does not provide any evidence that $p(P_t \rightarrow C_t) \equiv +1$ is true. Such an evidence needs to be ensured before a proof by modus ponens is performed. Modus ponens is based on the correctness of the material implication $p(P_t \rightarrow C_t) \equiv +1$ and assumes that $p(P_t \rightarrow C_t) \equiv +1$ is true, but does not provide any evidence of the correctness of this premise. In the following, we perform some investigations, measurements, thought experiments et cetera and have found that

$$P_t \equiv +1 \tag{204}$$

or true. Consequently, since $p(P_t \rightarrow C_t) \equiv +1$ or true and $P_t \equiv +1$ or true, we are allowed to conclude that

$$C_t \equiv +1 \tag{205}$$

or true. □

McGee describes modus ponens as follows:

"The rule of modus ponens, which tells us that from an indicative conditional If ϕ then ψ, together with its antecedent ϕ, on can infer ψ, is one of the fundamental principles of logic. "(see McGee, 1985, p. 462)

Another side of the same modus ponens is the fact that a true premise excludes a false conclusion. This fact is illustrated by table 18.

Table 18. A true premise excludes a false conclusion.

		Conclusion C_t		
		NO	YES	
Premise	YES	**+0**	**+1**	$p(P_t)$
P_t	NO	**+1**	**+1**	$p(\underline{P}_t)$
		$p(\underline{C}_t)$	$p(C_t)$	+1

According to the adopted form of modus ponens, it is not possible to reach a false conclusion while the premise is true as illustrated by table 18. No sound understanding of modus ponens can be built on an analysis which looks at only one side of this rule of inference. Taking into account all relevant aspects of modus ponens, another side of this rule of inference is self-evident and illustrated by table 19 (see also modus sine (see Barukčić, Ilija, 2019, p. 178-179)).

Table 19. Modus sine. Without false premise, no false conclusion.

		Conclusion C_t		
		NO	YES	
Premise	NO	**+1**	**+1**	$p(\underline{P}_t)$
P_t	YES	**+0**	**+1**	$p(P_t)$
		$p(\underline{C}_t)$	$p(C_t)$	+1

Let's leave it at that and end this topic by referring to Vann McGee's (see McGee, 1985) prima facie modus ponens counterexample (see Bledin, 2015, Katz, 1999, Lowe, 1987, Piller, 1996).

Example

We reconsider once again the secured knowledge or material implication **if** burning wax candle (P_t) **then** gaseous oxygen (C_t) is present (see table 20).

Table 20. Modus ponens and material implication.

		Gaseous oxygen C_t		
		YES	NO	
Burning candle	YES	**+1**	**+0**	$p(P_t)$
P_t	NO	**+1**	**+1**	$p(\underline{P}_t)$
		$p(C_t)$	$p(\underline{C}_t)$	+1

Our premise or material implication **if** burning wax candle (P_t) **then** gaseous oxygen (C_t) is present is true. Performing some observations, measurements, experiments et cetera we found that $P_t \equiv +1$ is true. Based on these facts we deduce that gaseous oxygen is present or that $C_t \equiv +1$ is true. However, based on the correctness of the material implication **if** burning wax candle (P_t) **then** gaseous oxygen (C_t) is present another conclusion is possible too (see table 21).

Table 21. A burning candle excludes the absence of gaseous oxygen

		Gaseous oxygen C_t		
		NO	YES	
Burning candle	YES	**+0**	**+1**	$p(P_t)$
P_t	NO	**+1**	**+1**	$p(\underline{P}_t)$
		$p(\underline{C}_t)$	$p(C_t)$	+1

In other words, a burning candle excludes the absence of gaseous oxygen. Such a conclusion is another straightforward consequence of modus ponens.

4. Discussion

Researchers who are interested in investigating the relationship between an event A_t (i.e. exposure, risk factor, condition et cetera) and an event B_t (i.e. disease, outcome, conditioned et cetera) need to consider several aspects. Based on the research question, a hypothesis is defined and then decided which study design is best suited to answer that question. The chosen study design, experimental or observational based, has great impact on the quality of scientific knowledge and the conclusion drawn and how the investigation will be conducted. However, medical studies might have several limitations, including the design of a study too. It is useful to know that there is already empirical evidence of study design biases [29], [30], [31], [32], [33], [34] in medical studies. As generally known, experimental study design enables the investigator to control various aspects of the relationship between interventions and outcomes of interest. Observational studies (case report or case series, nested case-control, case-time-control studies, case-control studies, [35], case-crossover studies [36], cohort studies [37], cross-sectional studies [38], ecological studies [39]), are conducted in non-controlled environment and involve merely observing the events of interests without actually interfering or manipulating and therefore are non-experimental. However, the scientific results achieved need to be comparable and should be independent of the design of a study (experimental study design vs. non-experimental study design). Thus far, in order to address some investigative questions with respect to material implication, novel approaches in study design are needed. In last consequence, those who intend to investigate a material implication between random events by a medical study should try to routinely consider a study design which ensures an index of unfairness (IOU) as much as possible near or equal to IOU = 0. However, besides of such a study design, studies which are relatively quick and easy done where an investigator simply observes and no interventions are carried out can but need not permit a reliable distinction between a cause and an effect.

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5. Conclusion

Material implication, an important measure of relationship between events, has been re-formulated by the tools of probability theory and statistics.

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6. Patient consent for publication

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Conflict of interest statement

No conflict of interest to declare.

Private note

The definition section of a paper need not and does not necessarily contain new scientific aspects. Above all, it also serves to better understand a scientific publication, to follow every step of the arguments of an author and to explain in greater details the fundamentals on which a publication is based. Therefore, there is no objective need to force authors to reinvent a scientific wheel once and again unless such a need appears obviously factually necessary. The effort to write about a certain subject in an original way in multiple publications does not exclude the necessity simply to cut and paste from an earlier work, and has nothing to do with self-plagiarism. However, such an attitude cannot simply be transferred to the sections' introduction, results, discussion and conclusions et cetera.

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I was born October, 1st 1961 in Novo Selo, Bosnia and Herzegovina, former Yogoslavia. I am of Croatian origin. From 1982-1989 C.E., I studied human medicine at the University of Hamburg, Germany. Meanwhile, I am working as a specialist of internal medicine. My basic field of research since my high school days at the Wirtschaftsgymnasium Bruchsal, Baden Württemberg, Germany is the mathematization of the relationship between a cause and an effect valid without any restriction under any circumstances including the conditions of classical logic, probability theory, quantum mechanics, special and general theory of relativity, human medicine et cetera. I endeavour to investigate positions of quantum mechanics, relativity theory, mathematics et cetera, only insofar as these positions put into question or endanger **the general validity of the principle of causality**.

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