Causation **1** (2006), 5-10.

Causation and the law of independence.

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Abstract

Titans like Bertrand Russell or Karl Pearson warned us to keep our mathematical and statistical hands off causality and at the end David Hume too. Hume's philosophy has dominated discussion about causality for a long time. But more and more researchers are working hard on this field and trying to do it better. Much of the recent philosophical or mathematical writings on causation either addresses to Bayes networks, to the counterfactual approach to causality developed in detail by David Lewis, to Reichenbach's Principle of the Common Cause or to the Causal Markov Condition. None of this approaches to causation investigated the relationship between causation and the law of independence to a necessary extent. Only, may an effect occur in the absence of a cause? May an effect fail to occur in the presence of a cause? In so far, what does constitute the causal relation? On the other hand, if it is unclear what does constitute the causal relation, maybe we can answer the question, what does not constitute the causal relation. This publication will prove, that the law of independence defines causation to some extent **ex negativo**.

1. Introduction

Attempts to analyse the relationship between cause and effect in terms of probability theory are based on the fact that causes can raise (Patrick Suppes (1970)) or lower (Germund Hesslow (1976)) the probabilities of their effects. Probabilistic theories of causation offer a potential advantage over regularity theories (especially John Stuart Mill (1843), John Mackie (1974)), probabilistic approaches to causation are compatible with indeterminism.

2. Methods

According to David Hume, causes are followed by their effects or the asymmetry of causation (Hausman (1998)) is based on the temporal asymmetry between cause and effect. It is a remarkable fact that the definition of causation in terms of temporal asymmetry has a number of disadvantages. Firstly. The position "*post hoc, ergo propter hoc*" is known to be a logical fallacy. Secondly. The regularity approach to causation is known to be incompatible with quantum mechanics and Heisenberg's Uncertainty Principle. Thirdly. If the cause as such happens only before the effect, this rules out that the cause can happen after its effect. Thus, if causes only precede their effects in (space) time then it seems plausible, however, that there is no causation at all. Hence, because of the many well-known difficulties with the definition of causation in terms of temporal asymmetry another approach to causation is necessary.

3. Results

Causal investigation of the world around us using the tools of probability theory is often based on random variables. For a variety of reasons this appears to be reasonable. It is common to distinguish "the cause" as such and "a cause" (Mill (1843)). The first difficulty is to define, what is a cause, what is an effect. There are various, usually imprecise definitions of cause (f. e. Aristotle's doctrine of the four causes) and effect. In order to avoid certain major errors of definition, let us just talk about the cause or about the effect.

Theorem 1.

The determination of an effect by a cause and vice versa.
Let C_t denote the cause, something existing independently of human mind and consciousness, at the (space) time t. Let $E(C_t)$ denote the expectation value of the cause at the (space) time t. Let $E(C_t) \neq 0$. Let E_t denote the effect something other existing independently of human mind and consciousness, at the (space) time t. Let $E(E_t)$ denote the expectation value of the effect at the (space) time t. Let $E(C_t , E_t)$ denote the expectation value of cause and effect at the (space) time t. Let $\sigma(C_t , E_t)$ or $Cov(C_t , E_t)$ denote the covariance of cause and effect at the (space) time t. Then, according to the law of independence, one of the fundamental concepts in probability theory, the effect is independent from the cause and vice versa, if

$$\boldsymbol{\sigma (C_t , E_t) = Cov(C_t , E_t) = E(C_t , E_t) - (E(C_t) * E(E_t)) = 0.}$$

Proof of the Theorem 1.

$$\mathbf{+ C_t = + C_t}$$

The starting point of our proof is the identity of $\mathbf{+C_t = +C_t}$ (Barukčić 2006a, pp. 55-60, pp. 44-46). C_t is only itself, simple equality with itself, it is only self-related and unrelated to an other, it is distinct from any relation to an other, C_t contains nothing other but only itself. In this way, there does not appear to be any relation to an other, any relation to an other is removed, any relation to an other has vanished. Consequently, C_t is just itself and thus somehow the absence of any other determination. C_t is in its own self only itself and nothing else. In this sense, C_t is identical only with itself, C_t is thus just the 'pure' C_t. Let us consider this in more detail, C_t is not the transition into its opposite, the negative of C_t is not as necessary as the C_t itself, C_t is not confronted by its other. C_t is without any opposition or contradiction, is not against an other, is not opposed to an other, is identical only with itself and has passed over into pure equality with itself or C_t is without any local hidden variable.

But lastly, although identity and difference are somehow different, identity is not difference, identity is in its own self different. Thus, C_t immediately negates itself. C_t is at the same (space) time in its self-sameness different from itself and thus self-contradictory. Since $C_t = C_t$ it excludes at the same (space) time the other out of itself, it is C_t and it is nothing else, it is at the same (space) time not Not-C_t, C_t is thus non-being as the non-being of its other. In excluding its own other out of itself C_t is excluding itself in its own self. By excluding its other, C_t makes itself into the other of what it excludes from itself, or C_t makes itself into its own opposite, C_t is thus simply the transition of itself into its opposite. C_t is therefore alive only in so far as it contains such a contradiction within itself.

The non-being of its other is at the end the sublation of its other. This non-being is the non-being of itself, a non-being which has its non-being in its own self and not in another, each contains thus a reference to its other. Not-C_t is the pure other of C_t . But at the same (space) time, Not-C_t only shows itself in order to vanish, the other of C_t is not. C_t and Not-C_t are distinguished and at the same (space) time both are related to one and the same C_t , each is that what it is as distinct from its own other. Identity is thus to some extent at the same (space) time the vanishing of otherness. C_t is itself and its other, C_t has its determinateness not in an other, but in its own self. C_t is thus self-referred and the reference to its other is only a self-reference. On closer examination C_t therefore is, only in so far as its Not-C_t is, C_t has within itself a relation to its other. In other words, C_t is in its own self at the same (space) time different from something else or C_t is something. It is widely accepted that something is different from nothing, thus while $C_t = C_t$ it is at the same (space) time different from nothing or from **non -** C_t **.** From this it is evident, that the other side of the identity $C_t = C_t$ is the fact, that C_t cannot at the same (space) time be C_t and not C_t. In fact, if $C_t = C_t$ then C_t is not at the same (space) time non C_t. What emerges from this consideration is, therefore, even if $C_t = C_t$ it is a self-contained opposition. C_t is only in so far as C_t contains this contradiction within it, C_t is inherently self-contradictory. C_t is thus only as the other of the other. In so far, C_t includes within its own self its own non-being, a relation to something else different from its own self. Thus, C_t is at the same (space) time the unity of identity with difference. C_t is itself and at the same (space) time its other too, C_t is thus contradiction. Difference as such imply contradiction because it unites sides which are, only in so far as they are at the same (space) time not the same. C_t is only in so far as the other of C_t, the non-C_t is. C_t is thus that what it is only through the other, through the non-C_t, through the non-being of itself. Thus we obtain

$$\mathbf{+C_t - C_t = 0.}$$

+C_t and **-C_t** are negatively related to one another and both are indifferent to one another, C_t is separated in the same relation. C_t is itself and its other, it is self-referred, its reference to its other is thus a reference to itself, its non-being is thus only a moment in it. C_t is in its own self the opposite of itself, it has within itself the relation to its other, it is a simple and self-related negativity. Each of them are determined against the other, the other is in and for itself and not as the other of an other. C_t is in its own self the negativity of itself. C_t therefore is, only in so far as its non-being is and vice versa. Non - C_t therefore is, only in so far as its non-being is, both are through the non-being of its other, both as opposites cancel one another in their combination.

Further, the identity of $C_t = C_t$ is an identity over time. Time as such involves in a very general way something like an alteration. C_t undergoes alteration, it goes outside itself. In general, any alteration of C_t, the cause, raises subtle problems. How can the cause remain the same and yet change? If C_t changes, must there be a cause for this change or is an uncaused change possible? Is it extremely implausible to deny caused change? Thus, if $C_t = C_t$ and if C_t changes too, then C_t must at the same (space) time at least be non-identical to itself. In so far, C_t must include a difference within itself or to say it more mathematically, there must be an expectation value of C_t. According to Kolmogorov it holds true that **"If *x* and *y* are equivalent then *E(x) = E(y)* ."** (Kolmogorov 1956, p. 39). Thus we obtain the next equation

$$\mathbf{E(C_t) = E (C_t).}$$

If $C_t = C_t$ then $E(C_t) = E(C_t)$. This does not mean that it must hold true that $C_t = E(C_t)$! If it is only that $C_t = C_t$, how can an advance to something different be made? Let us assume, that the cause C_t is not alone. In other words, it is true that

$$E(C_t) \mathbf{*\ 1} = E (C_t).$$

Let **$E(E_t) = E(E_t)$.** Let **$E(E_t) \neq 0$**, thus **$E(E_t)/E(E_t) = 1$**. It is $E(E_t) = E(E_t)$ and $E(C_t) = E(C_t)$ but both are not one. The self-identity of both is thus the indifference of each towards the other which is distinguished from it. In the same relation, both are rigidly held as separated., both have a separate existence and are without any relation to an other. In this case, a cause has no relation to an effect, nothing changes by the cause, effect E_t is like it is, thus we obtain

$$E(C_t) * \mathbf{(E(E_t) / E(E_t))} = E(C_t)$$

or

$$E(C_t) * E(E_t) = E(C_t) * E(C_t)$$

Each of both stands isolated from each other, is separated from each other, each is only on its own. By this separation of one from the other, both are related not to one another, each is valid on its own and without any respect to an other. In so far, according to Kolmogorov, it is " **$E(X Y)$** = ... = $E(X E(Y))$ **$= E(X) * E(Y)$** " (Kolmogorov 1956, p. 60). Thus we obtain

$$E(C_t, E_t) = E(C_t) * E(E_t)$$

or

$$E(C_t, E_t) - E(C_t) * E(E_t) = 0$$

However, in general, if the effect is independent from the cause and vice versa or if the cause is independent from itself and equally determining itself, that is to say, if the probability of the cause p(C) is either p(C) = 1 or p(C) = 0, then

no causal relationship

between cause and effect can be proofed or established by the tools of probability theory or statistics in this case. In so far, it holds true that

$$\boxed{\boldsymbol{\sigma(C_t, E_t) = Cov(C_t, E_t) = E(C_t, E_t) - (E(C_t) * E(E_t)) = 0.}}$$

Q. e. d.

4. Discussion

If cause and effect are independent from each other, if the cause is only for itself and without any relation to an other, if the cause is independent from itself and equally **determining** itself (Barukčić 2006a, p.44), then it is true that

$$\boldsymbol{\sigma (C_t , E_t) = Cov(C_t , E_t) = E(C_t ,E_t) - (E (C_t) * E(E_t)) = 0.}$$

If $\sigma(C_t , E_t) \neq 0$, this not a clear proof by the tools of probability theory or statistics that there is a causal relationships between C_t and E_t. We can only state that is not possible to extract causal relationships from data with the tools of probability theory or statistics, if $\sigma(C_t , E_t) = 0$. In so far, the law of independence, one of the fundamental laws in nature, statistics and probability theory is valid for the relationship between cause and effect too. If the effect at the same (space) time is independent from the cause and vice versa, if the cause at the same (space) time independent from the effect, then it holds true that

$$\boldsymbol{\sigma (C_t , E_t) = Cov(C_t , E_t) = E(C_t , E_t) - (E (C_t) * E(E_t)) = 0.}$$

Under this circumstances it is difficult to proof or establish a causal relationship by the tools of probability theory or statistics. As long as $\sigma(C_t , E_t) \neq 0$, it appears to be possible to use the tools of probability theory or statistics to extract causal relationships from data. Causation is in so far to some extent the other of independence and at the same (space) time an absolutely necessary part of independence (Barukčić 2006a, p. 44) too, independence defines thus causation to some extent **ex negativo.**

Acknowledgement

This publication is part of my work presented by me at the XXIIIrd International Biometric Conference scheduled from July 16-21, 2006 in Montréal, Canada. In particular, I am extremely grateful to the Scientific Program Committee of the XXIIIrd International Biometric Conference and especially to Alain Vandal for the trust and support provided by reviewing and accepting my paper entitled: “New Method for Calculating Causal Relationships” for presentation at the XXIIIrd International Biometric Conference in Montréal, Canada. I am very grateful to Agnes M. Herzberg and M.E. Thompson.

References

Barukčić, Ilija. (1989). *Die Kausalität*. 1. Ed. Hamburg: Wissenschaftsverlag.

Barukčić, Ilija. (2006a). *Causality. New Statistical Methods*. Hamburg: Books on Demand. pp. 488.

Barukčić, Ilija. (2006b). *New Method For Calculating Causal Relationships*, Montréal: XXIII International Biometric Conference, July 16 - 21 2006.

Eells, Ellery. (1991). *Probabilistic Causality*. Cambridge, U.K.: Cambridge University Press.

Einstein, Albert. (1908a). "Über das Relativitätspnnzip und die aus demselben gezogenen Folgerungen," *Jahrbuch der Radioaktivität und Elektronik* 4: 411-462.

Einstein, Albert. (1908b). "Berichtigungen zu der Arbeit: Über das Relativitätspnnzip und die aus demselben gezogenen Folgerungen," *Jahrbuch der Radioaktivität und Elektronik* 5: 98-99.

Hausman, Daniel. (1998). *Causal Asymmetries*. Cambridge: Cambridge University Press.

Hesslow, Germund. (1976). "Discussion: Two Notes on the Probabilistic Approach to Causality," *Philosophy of Science* 43: 290 - 292.

Hegel, Georg Wilhelm Friedrich (1998). *Hegel's science of logic*, Edited by H. D. Lewis, Translated by A. V. Miller. New York: Humanity Books.

Hume, David. (1748). *An Enquiry Concerning Human Understanding*.

Kolmogorov, A. N. (1933). *[Grundbegriffe der Wahrscheinlichkeitsrechnung, 1933] Foundations of the Theory of Probability,* Transl. Nathan Morrison, sec. Ed. Repr. 1956, New York: Chelsea Publishing Company.

Mackie, John. (1974). *The Cement of the Universe*. Oxford: Clarendon Press.

Mill, John Stuart. (1843). *A System of Logic, Ratiocinative and Inductive*. London: Parker and Son.

Noordhof, Paul. (1999). "Probabilistic Causation, Preemption and Counterfactuals," *Mind* 108**:** 95 - 125.

Pearl, Judea. (2000). *Causality: Models, Reasoning, and Inference*. Cambridge: Cambridge University Press.

Reichenbach, Hans. (1956). *The Direction of Time*. Berkeley and Los Angeles: University of California Press.

Salmon, Wesley. (1980). "Probabilistic Causality," *Pacific Philosophical Quarterly* 61**:** 50 - 74.

Spirtes, Peter, Clark Glymour, and Richard Scheines. (2000). *Causation, Prediction and Search*, Second Edition. Cambridge, MA: M.I.T. Press.

Suppes, Patrick. (1970). *A Probabilistic Theory of Causality*. Amsterdam: North-Holland Publishing Company.

Published: December 15th, 2006.
Revision: February 12th, 2007.